OPTIMAL KINEMATIC SYNTHESIS OF A 4-BAR LINKAGE FOR N-POSITION LINK GUIDING

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By

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to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1971

CERTIFICATE

This is to certify that the dissertation entitled "Optimal Kinematic Synthesis of a 4-Bar Linkage for N-Position Link Guiding" by Mr. Subodh Kumar Chaturvedi has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

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NOMENCLATURE

Notations	Terms
X	Magnitude of particular link or angle
O_A	Pivot point of link 2
O_B	Pivot point of link 4
X_1	Base link
X_2	Input crank
X_3	Coupler
X_4	Output link
X_5	Link fixed on coupler determining coupler point
X_6	Distance between O_A and origin
X_7	Input angle
X_8	Inclination of link 1 from the horizontal
X_9	Included angle between link 5 and link 3
X_{10}	Inclination of link 6 from the horizontal
η	Inclination of link 3 from the horizontal
$\bar{D}$	Design vector
x_P	X-coordinate of coupler point obtained from synthesized mechanism
y_P	Y-coordinate of coupler point obtained from synthesized mechanism
$E(\bar{D})$	Total error function
$E_c(\bar{D})$	Coordinate matching error
$E_o(\bar{D})$	Guide angle matching error

σ_1	Weighting factor for coordinate matching error
σ_2	Weighting factor for angle matching error
D	Diagonal length joining O_B and one end of link 2
λ	Included angle between diagonal D and link 1
β	Included angle between D and link 4
δ	Exterior angle between link 4 and link 1
X_{3x}	Horizontal projection of link 3
X_{3y}	Vertical projection of link 3
X_{min}	Minimum possible link length
X_{max}	Maximum possible link length
$R_A, \bar{X}_A$	Feasible region for O_A defined by radius R_A around the centre defined by vector $\bar{X}_A$
$R_B, \bar{X}_B$	Feasible region for O_B defined by radius R_B around the centre defined by vector $\bar{X}_B$
$\measuredangle X_A$	Angular position of $\bar{X}_A$
$\measuredangle X_B$	Angular position of $\bar{X}_B$
$\theta(X), W(x)$	Force $W(x)$ applied at coupler point at an angle $\theta(x)$ with horizontal
$\bar{K}_2$	Force vector in link 2
$\bar{K}_4$	Force vector in link 4
$\bar{T}_2$	Moment vector in link 2
K_2	Maximum allowable strength in link 2
K_4	Maximum allowable strength in link 4
T_2	Maximum allowable torque in link 2
μ	Transmission angle
$f(\bar{X})$	Objective function
$g_i(\bar{X})$	Inequality constraints

r	Fiacco-McCormick constraint multiplier
C	Factor by which value of 'r' is reduced
$p(\bar{D})$	Secondary function
$\bar{G}(X)$	Gradient vector
α	Optimum step length
Z_i	Vector defining i^{th} link
t_i	Incremental rotation of crank in i^{th} position from its original position
u_i	Incremental rotation of coupler in i^{th} position from its original position
v_i	Incremental rotation of output link in i^{th} position from its original position

Notations which are not given in the above list have been explained at the place used.

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ABSTRACT

The design of 4-bar mechanisms for finitely separated positions of coupler guide has been considered. An analytical technique is formulated to synthesize for maximum of 3 or 5 possible positions, depending whether the positions of the crank are coordinated or non-coordinated. When certain design and side constraints are imposed, an optimum solution is sought where inequality constraints are handled through Penalty Function.

CHAPTER I

INTRODUCTION

Kinematic synthesis techniques have been developed and refined to the point where, at least in the planar area, the kinematician has a fairly good idea of the type of motion that can be obtained with a 4-bar linkage. Freudenstein [1-4] in his earlier papers, provided the background necessary to encourage the widespread interest in analytical kinematics that followed. With the invent of high speed computers the trend has been to replace conventional graphical techniques by sophisticated analytical methods where accuracy of design and economy of time are of prime importance. The present work has been carried on similar lines. All solutions have been obtained on IBM 7044.

1.1. Link Guiding Problem - Definition :

Graphical solutions [5] of the 5 position problem with non-coordinated crank is quite well known. When the positions of the coupler are to be coordinated with crank, the maximum number of positions to be specified is reduced to 3.

In all the above cases we get an exact solution of the 4-bar linkage. When certain other constraints such as maximum and minimum transmission angle, the approximate location of pivot points etc. are imposed on the problem, we may not in general, arrive at any exact solution, even

for the above cases. What we can do at most is to find the best solution which will give a mechanism that satisfies all the constraints and at the same time gives the minimum error for the positions when the crank is rotated through desired angles.

In the present dissertation, N-positions of the coupler and corresponding rotations of the crank are specified. In this case no closed form solution can be obtained. The best solution is arrived at by making a mathematical model and then seeking an optimal choice. For the given positions an analytical solution is sought for any 3 out of N-positions and this becomes the starting solution for iterations. In this method, the mathematical programming problem with constraints is transformed into a sequence of unconstrained minimization problems.

1.2. Review of the Earlier Work :

The concept of optimization has been applied to a number of problems in kinematic synthesis in recent years. The work of Kiss [6] involves a generalized approach in terms of basic and characteristic motion criteria. Dynamic programming techniques are applied to solve the function generation problem. To solve function generation problem by 4-bar linkages, Garrett and Hall [7] randomly generated a large number of linkages and this was followed by the choice of the best solution on the basis of minimum root mean square error. Timko [8] also attacked the function generation problem

in the same way. Fox and Willmert [9] have tackled the curve-tracing problem in an elegant way. The problem of synthesizing a 4-bar linkage is presented as mathematical programming problem where inequality constraints are handled through Penalty Function. Fox and Moore [10] have solved the same problem, the difference being that they have assumed the error function as a 'continuous' function while Willmert took it as a 'precision' point problem. The approach outlined by Han [11] holds considerable promise for the application of dynamic programming to a wide range of design problems in kinematic synthesis. Gustavson [12] applies weighting factors to three necessary design criteria to sort a large number of solutions, for point synthesis in terms of Burmester Curves for 4-finitely separated positions. Another unique approach for point synthesis in terms of damped least square iteration is given by Lewis and Gyory [13] . Eschenbach and Tesar [14] have solved the problem for 4-generalized coplanar positions. The constraints are obtained from pairs of corresponding points on the generalized Burmester Curves. These linkages are sorted by means of zones and given design criteria based on weighting factors applied to various necessary design requirements.

The present dissertation is an attempt to give an unified approach to point tracing, function generation and link guiding problems of 4-bar linkages. The approach is believed to be unique in the sense that this is the first attempt of this kind and most general in the sense that it seeks

the best design, for any given criterion, for, not only the well-known 3,4 or 5 position problems, but also for any finite number of given positions. 3, 4 or 5 point problems in point tracing, function generation or link guiding are but the particular cases of the approach presented.

In Chapter II analytical approach to the problem, for limited number of positions, using complex numbers has been discussed. It has also been shown that how the maximum number of 5 positions for non-coordinated crank reduces to 3, when crank is required to occupy the assigned positions.

In Chapter III mathematical model of the problem in terms of objective function and design constraints is developed. Chapter IV deals with the solution algorithm. Few numerical examples have been included in Chapter V. Discussions and Conclusions have been given in Chapter VI.

CHAPTER II

ANALYTICAL APPROACH TO THE PROBLEM

The method of complex numbers is applied for the kinematic synthesis of 4-bar link guiding problem. It is shown that upto 5 positions, closed form solution can be obtained for non-coordinated crank problem. In this case the designer has no control over the design. In 4 positions designer has the control over the design by varying arbitrary parameters. The method has been programmed for automatic ditigal computation on IBM 7044, and is included in Appendix.

2.1. Coordinate System and Notations :

In Figure 1 X-Y, is the coordinate system with 'O' as its origin and P as the coupler point with (X_P, Y_P) as its coordinates. The linkage as shown consists of link 1, 2, 3, 4 and 5 having lengths a, b, c, d and l respectively and angular positions α, θ, β, ϕ, ψ.

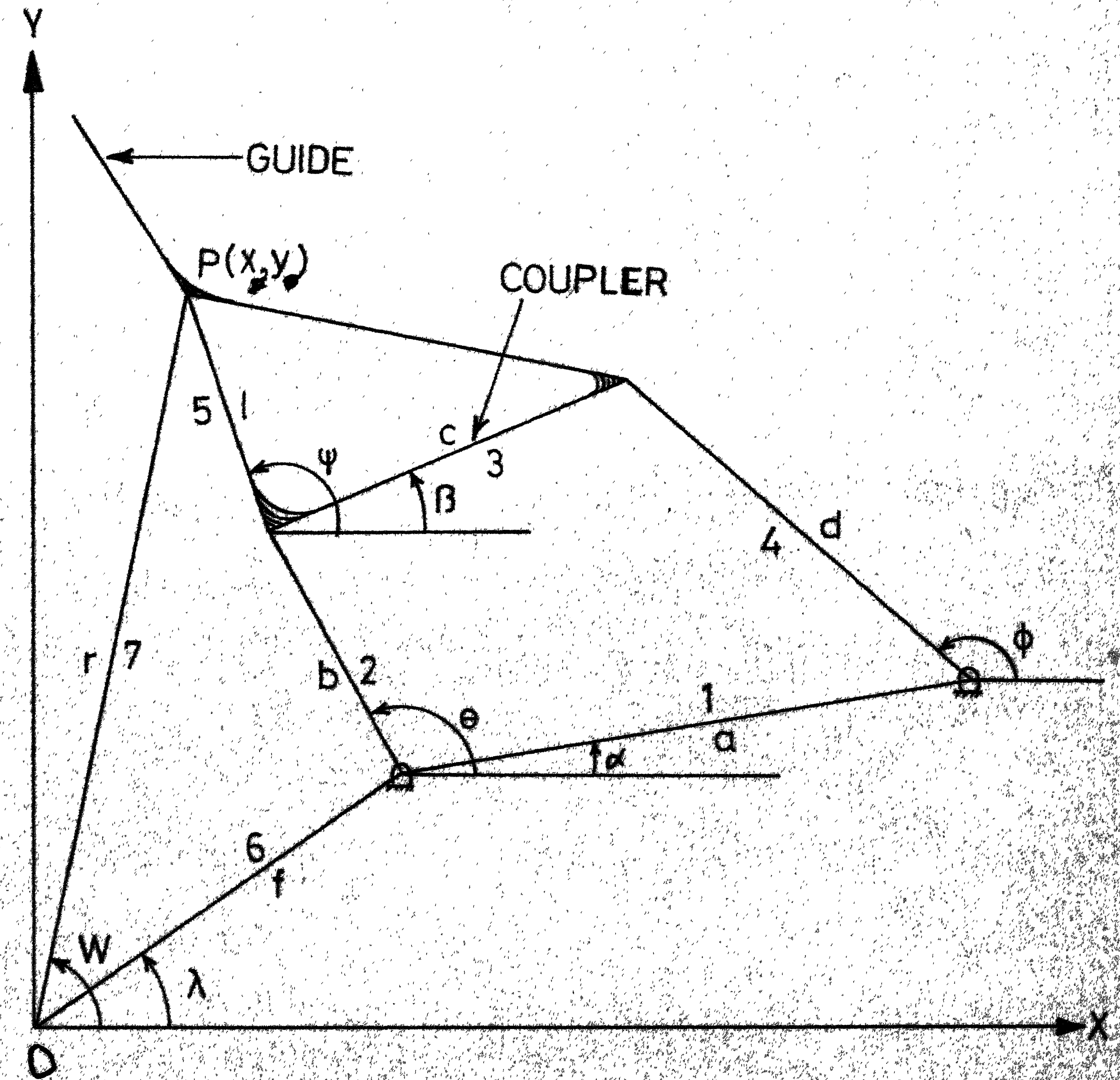
2.2. Position Equations :

Loop 1 consists of links 1, 2, 3 and 4 and position equation can be written as

$$b\,e^{j\theta} + c\,e^{j\beta} = a\,e^{j\alpha} + d\,e^{j\phi} \tag{2.1}$$

Loop 2 consists of links 6, 2, 5 and 7 and position equation can be written as

$$f\,e^{j\lambda} + b\,e^{j\theta} + l\,e^{j\psi} = r\,e^{j\omega} \tag{2.2}$$



FOUR BAR LINKAGE TERMINOLOGY

LOOP 1 CONSISTS OF LINKS 1 2 3 & 4

LOOP 2 CONSISTS OF LINKS 2 5 7 & 6

FIGURE 1

Since the motion of each link is to be expressed in terms of its rotation from an unspecified starting position, it is convenient to write

$$\theta = \theta_1 + t$$
$$\beta = \beta_1 + u$$
$$\psi = \psi_1 + u$$
$$\text{and } \phi = \phi_1 + v$$

Writing equations (2.1) and (2.2) as

$$b\,e^{j\theta_1}\;e^{jt} + c\,e^{j\beta_1}\;e^{ju} = a\,e^{j\alpha} + d\,e^{j\phi_1}\,e^{jv} \qquad (2.3)$$

$$f\,e^{j\lambda} + b\,e^{j\theta_1}\;e^{jt} + l\,e^{j\psi_1}\;e^{ju} = r\,e^{jw} \qquad (2.4)$$

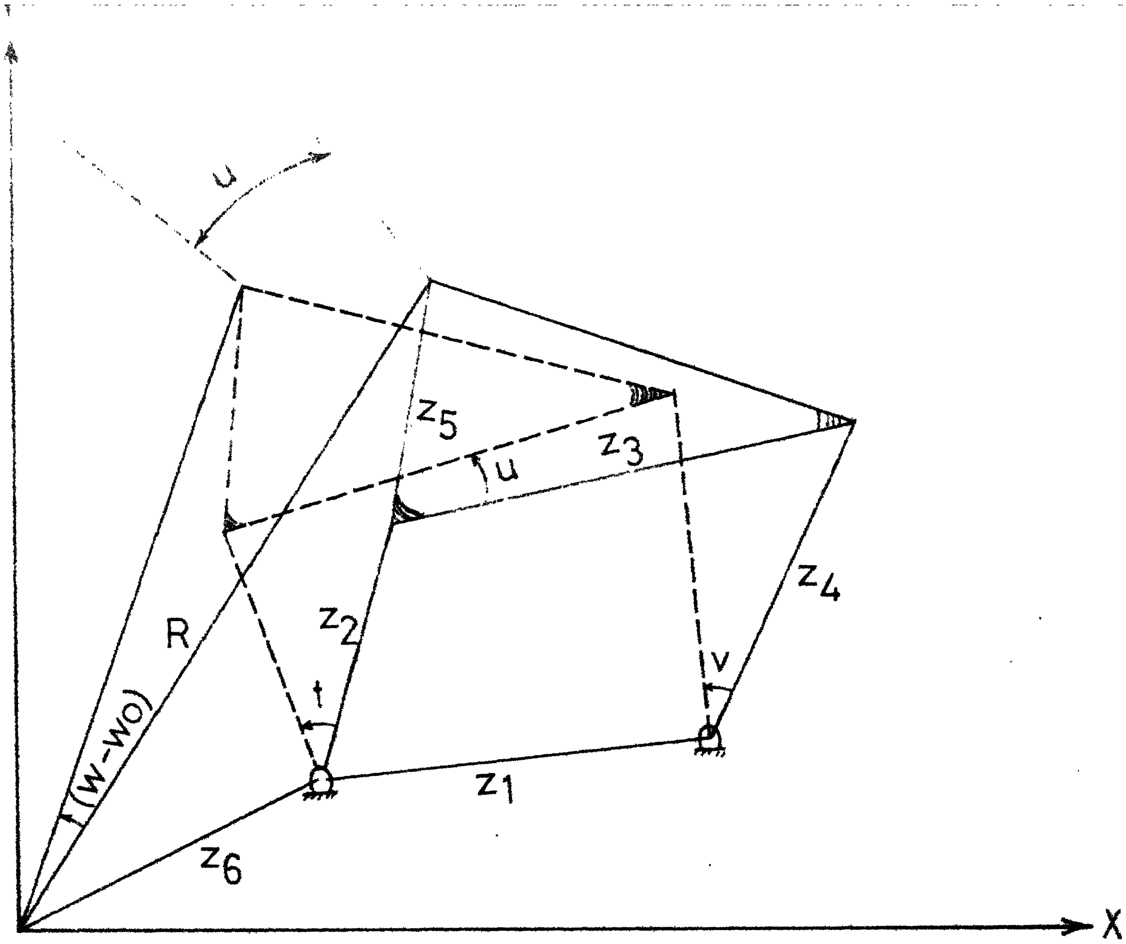
Now if we define vectors as

$$Z_1 = a\,e^{j\alpha}$$
$$Z_2 = b\,e^{j\theta_1}$$
$$Z_3 = c\,e^{j\beta_1}$$
$$Z_4 = d\,e^{j\phi_1}$$
$$Z_5 = l\,e^{j\psi_1}$$
$$Z_6 = f\,e^{j\lambda}$$
$$R = r\,e^{jw}$$

Hence the position equations can be written as

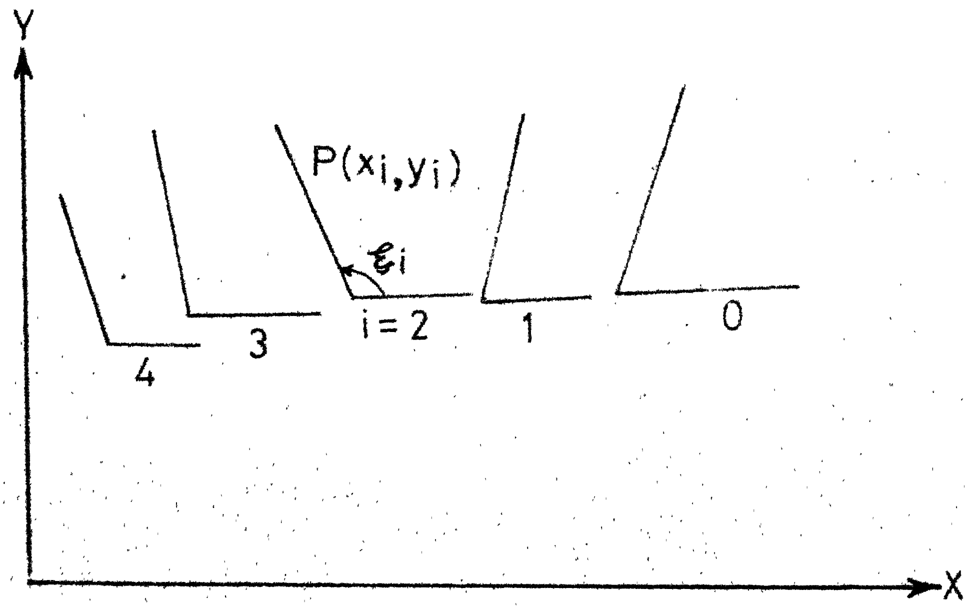
$$Z_2\,e^{jt} + Z_3\,e^{ju} - Z_4\,e^{jv} = Z_1 \qquad (2.5)$$

$$Z_6 + Z_2\,e^{jt} + Z_5\,e^{ju} = R \qquad (2.6)$$



THE FOUR-BAR LINKAGE IN ITS ORIGINAL POSITION (DARK) AND A DISPLACED (DASHED) POSITION

FIG. 2



VARIOUS POSITIONS OF GUIDE

FIG. 3

2.3. Use of Position Equation for Synthesis :

In Figure 3 various positions of guide are given. To specify the location of a guide at any position, its orientation and the coordinates of any point on it, say coupler point, must be known. Since the guide is fixed on the coupler, its orientation in the i^{th} position from original position would be u_i, as that for coupler. Hence the problem is defined by specifying coordinates of coupler point (X_i, Y_i) and u_i.

In the form of equation

$$\xi_i - \xi_o = u_i \qquad (i = 1,2,\ldots,n)$$

Therefore, the problem for (n+1) positions could be defined as

$$Z_2\, e^{jt_i} + Z_3\, e^{ju_i} - Z_4\, e^{ju_i} = Z_1 \qquad (i = 0,1,\ldots,n) \quad (2.7)$$

$$Z_6 + Z_2\, e^{jt_i} + Z_5\, e^{ju_i} = R_i \qquad (i = 0,1,\ldots,n) \quad (2.8)$$

Solution of equations (2.7) and (2.8) gives the required synthesis.

2.4. Determination of Maximum Number of Coupler Positions for Coordinated Crank Problem :

In this case the guide is required to occupy various specified positions corresponding to given angular positions of input crank.

Known quantities for (n+1) positions are

R_i	for	$(i = 0,1,\ldots\ldots,n)$	$= n+1$
t_i	for	$(i = 1,2,\ldots\ldots,n)$	$= n$

and unknown quantities are

$$v_i \quad \text{for} \quad (i = 1,2,\ldots\ldots,n) \quad = n$$

$$Z_1,\ Z_2,\ Z_3,\ Z_4,\ Z_5 \text{ and } Z_6$$

Since each vector requires two real quantities for its specification, its magnitude and direction, the six vectors give 12 unknowns. Hence total number of unknowns are 12 + n.

Each complex equation gives two real equations, hence total number of available real equations, referring to equations (2.7) and (2.8), are 4(n+1). Hence the maximum possible positions for which the problem could be solved is

$$4n + 4 = 12 + n$$

$$\text{or} \quad n_{max} = 2$$

Therefore maximum number of positions for this case would be 3 (i = 0,1,2).

2.5. Determination of Maximum Number of Coupler Positions for Non-coordinated Crank Problem :

In this case number of known quantities are

$$u_i \quad \text{for} \quad (i = 1,2,\ldots\ldots,n) \quad = \quad n$$

and number of unknown quantities are

$$v_i \quad \text{for} \quad (i = 1,2,\ldots\ldots,n) \quad = \quad n$$

$$t_i \quad \text{for} \quad (i = 1,2,\ldots\ldots,n) \quad = \quad n$$

Six vectors Z_1, $Z_2, \ldots, Z_6$ giving 12 unknowns. Hence the maximum possible positions for which the problem could be solved is

$$4n + 4 = 12 + 2n$$

$$\text{or } n_{max} = 4$$

Therefore maximum number of positions for this case would be 5 ($i = 0, 1, \ldots, 4$).

Now onwards, 3 position problem shall refer to coordinated crank problem and 4 or 5 position problem to non-coordinated crank problem.

2.6. Synthesis of Three Position Problem :

As explained in section 2.3 we can write equations similar to equations (2.7) and (2.8) for 3 positions

Loop 1 :

$$Z_2 + Z_3 - Z_4 = Z_1 \tag{2.9}$$

$$Z_2\, e^{jt_1} + Z_3\, e^{ju_1} - Z_4\, e^{jv_1} = Z_1 \tag{2.10}$$

$$Z_2\, e^{jt_2} + Z_3\, e^{ju_2} - Z_4\, e^{jv_2} = Z_1 \tag{2.11}$$

Loop 2 :

$$Z_6 + Z_2 + Z_5 = r_0\, e^{jw_0} \tag{2.12}$$

$$Z_6 + Z_2\, e^{jt_1} + Z_5\, e^{ju_1} = r_1\, e^{jw_1} \tag{2.13}$$

$$Z_6 + Z_2\, e^{jt_2} + Z_5\, e^{ju_2} = r_2\, e^{jw_2} \tag{2.14}$$

It is clear from above equations that there are 14 unknowns ,2 for angles v_1, v_2 and 12 for 6 vectors, but we have only 12 real equations at our disposal. Therefore in order to get a nontrivial solution we can keep v_1 and v_2 arbitrary.

First we solve equations (2.12 - 2.14) for Z_2, Z_5 and Z_6 and then solving (2.9) - (2.11), we find Z_1, Z_3 and Z_4. A careful examination of the above equations reveals that no solution of the problem exists if ($t_1 = u_1$ and $t_2 = u_2$) and/or ($u_1 = v_1$ and $u_2 = v_2$).

By subtracting equation (2.12) from equation (2.13) and (2.14), we get two equations with no Z_6 term

$$a_1 Z_2 + b_1 Z_5 = c_1 \tag{2.15}$$

$$a_2 Z_2 + b_2 Z_5 = c_2 \tag{2.16}$$

where

$$\left.\begin{aligned} a_i &= 1 - e^{jt_i} && (2.17)\\ b_i &= 1 - e^{ju_i} && (2.18)\\ c_i &= r_0 e^{jw_0} - r_i e^{jw_i} && (2.19)\end{aligned}\right\} \quad (i=1,2)$$

Multiplying equation (2.15) by $(-\frac{a_2}{a_1})$ and adding equation (2.16) :

$$b_2' Z_5 = c_2' \tag{2.20}$$

where

$$b_2' = b_2 - \frac{b_1 a_2}{a_1} \tag{2.21}$$

$$c_2' = c_2 - \frac{c_1 a_2}{a_1} \tag{2.22}$$

Knowing Z_5 from equation (2.20), we can find Z_2 from equation (2.15) and Z_6 from equation (2.12).

Writing equations (2.9)-(2.11) as

$$Z_1 + Z_4 - Z_3 = Z_2 \tag{2.23}$$

$$Z_1 + Z_4\, e^{jv_1} - Z_3\, e^{ju_1} = Z_2\, e^{jt_1} \tag{2.24}$$

$$Z_1 + Z_4\, e^{jv_2} - Z_3\, e^{ju_2} = Z_2\, e^{jt_2} \tag{2.25}$$

These equations are alike (2.12)-(2.14), only difference being

$$a_i = 1 - e^{jv_i} \tag{2.26}$$

$$b_i = e^{ju_i} - 1 \qquad (i = 1,2) \tag{2.27}$$

$$c_i = Z_2 - Z_2\, e^{jt_i} \tag{2.28}$$

and Z_6, Z_2 and Z_5 are replaced by Z_1, Z_4 and Z_3.

2.7. Synthesis of Four Position Problem :

We can write position equations for this problem as

Loop 1 :

$$Z_1 + Z_4 - Z_3 = Z_2 \tag{2.29}$$

$$Z_1 + Z_4\, e^{jv_1} - Z_3\, e^{ju_1} = Z_2\, e^{jt_1} \tag{2.30}$$

$$Z_1 + Z_4\, e^{jv_2} - Z_3\, e^{ju_2} = Z_2\, e^{jt_2} \tag{2.31}$$

$$Z_1 + Z_4\, e^{jv_3} - Z_3\, e^{ju_3} = Z_2\, e^{jt_3} \tag{2.32}$$

Loop 2 :

$$Z_6 + Z_2 + Z_5 = r_0\, e^{jw_0} \tag{2.33}$$

$$Z_6 + Z_2\, e^{jt_1} + Z_5\, e^{ju_1} = r_1\, e^{jw_1} \tag{2.34}$$

$$Z_6 + Z_2\, e^{jt_2} + Z_5\, e^{ju_2} = r_2\, e^{jw_2} \tag{2.35}$$

$$Z_6 + Z_2\, e^{jt_3} + Z_5\, e^{ju_3} = r_3\, e^{jw_3} \tag{2.36}$$

It is clear from above equations that there are 18 unknowns, 6 for angles t_1, t_2, t_3, v_1, v_2 and v_3 and 12 for 6 vectors, but we have only 16 real equations at our disposal. Hence to get nontrivial solution we keep 2 quantities as arbitrary.

First we solve equations (2.33)-(2.36) for Z_2, Z_5 and Z_6. In this set we have 9 unknowns, 3 for t_1, t_2, t_3 and six for 3 vectors Z_2, Z_5 and Z_6 but only 8 real equations. Hence we choose t_1 arbitrary and express t_2 and t_3 in terms of t_1.

Proceeding as in section 2.6 we have :

$$a_1\, Z_2 + b_1\, Z_5 = c_1 \tag{2.37}$$

$$a_2\, Z_2 + b_2\, Z_5 = c_2 \tag{2.38}$$

$$a_3\, Z_2 + b_3\, Z_5 = c_3 \tag{2.39}$$

where $$a_i = 1 - e^{jt_i} \tag{2.40}$$

$$b_i = 1 - e^{ju_i} \qquad (\, i = 1, \ldots, 3\,) \tag{2.41}$$

$$c_i = r_0\, e^{jw_0} - r_i\, e^{jw_i} \tag{2.42}$$

And,

$$b_2' \, Z_5 = c_2' \tag{2.43}$$

$$b_3' \, Z_5 = c_3' \tag{2.44}$$

where $$b_i' = b_1 - \frac{b_i a_1}{a_i} \qquad (i = 2,3) \tag{2.45}$$

$$c_i' = c_1 - \frac{c_i \, a_1}{a_i} \tag{2.46}$$

Using equations (2.43) - (2.44) we can find characteristic equation.

$$\frac{c_2'}{b_2'} = \frac{c_3'}{b_3'} \tag{2.47}$$

$$\text{or } (c_1 - \frac{c_2 a_1}{a_2}) \, (b_1 - \frac{b_3 a_1}{a_3}) = (c_1 - \frac{c_3 a_1}{a_3}) \, (b_1 - \frac{b_2 a_1}{a_2}) \tag{2.48}$$

$$\text{or } (c_3 b_2 - c_2 b_3) a_1^2 + a_1 a_2 \, (c_1 b_3 - c_3 b_1) + a_1 a_3 (c_2 b_1 - c_1 b_2) = 0 \tag{2.49}$$

Defining d_1, d_2 and d_3 as

$$d_1 = (c_3 b_2 - c_2 b_3) \, a_1^2 \tag{2.50}$$

$$d_2 = (c_1 b_3 - c_3 b_1) \, a_1 \tag{2.51}$$

$$d_3 = (c_2 b_1 - c_1 b_2) \, a_1 \tag{2.52}$$

$$d_4 = d_1 + d_2 + d_3 \tag{2.53}$$

Writing equation (2.49) as

$$d_4 - d_2 \, e^{jt_2} = d_3 \, e^{jt_3} \tag{2.54}$$

Taking complex conjugate of both sides

$$\bar{d}_4 - \bar{d}_2 e^{-jt_2} = \bar{d}_3 e^{-jt_3} \tag{2.55}$$

Multiplying equations (2.54) and (2.55) and rearranging terms

$$d_4\bar{d}_4 + d_2\bar{d}_2 - d_3\bar{d}_3 - d_2\bar{d}_4 e^{jt_2} - \bar{d}_2 d_4 e^{-jt_2} = 0 \tag{2.56}$$

Defining d_5 as

$$d_5 = d_4\bar{d}_4 + d_2\bar{d}_2 - d_3\bar{d}_3 \tag{2.57}$$

Equation (2.56) can be written as

$$d_2\bar{d}_4 e^{2jt_2} - d_5 e^{jt_2} + \bar{d}_2 d_4 = 0 \tag{2.58}$$

This is a quadratic equation in e^{jt_2} and its two roots are given as

$$e^{jt_2} = \frac{d_5 \pm (d_5^2 - 4d_2\bar{d}_2 d_4\bar{d}_4)^{1/2}}{2d_2\bar{d}_4} \tag{2.59}$$

Knowing e^{jt_2}, e^{jt_3} can be found from equation (2.54) as

$$e^{jt_3} = \frac{d_4 - d_2 e^{jt_2}}{d_3} \tag{2.60}$$

Knowing e^{jt_2} and e^{jt_3} we can solve for Z_6, Z_2 and Z_5 using equations (2.43), (2.37) and (2.33).

The set of equations (2.29) - (2.32) is alike (2.33) - (2.36). There are 9 unknowns, 3 for v_1, v_2 and v_3 and 6 for 3 vectors Z_1, Z_3 and Z_4. Hence v_1 is chosen arbitrary.

Where,

$$a_i = 1 - e^{jv_i} \qquad (2.61)$$

$$b_i = e^{ju_i} - 1 \qquad (i = 1,\ldots,3) \qquad (2.62)$$

$$c_i = Z_2 - Z_2\, e^{jt_i} \qquad (2.63)$$

and we can proceed in the same way for solving Z_1, Z_4 and Z_3.

A careful examination of the above equations reveals that no solution exists if ($t_1 = u_1$, $t_2 = u_2$ and $t_3 = u_3$) and/or ($u_1 = v_1$, $u_2 = v_2$ and $u_3 = v_3$).

CHAPTER III

MATHEMATICAL FORMULATION

In this section, the mathematical model of the 4-bar linkage is formulated. The term 'mathematical model' means the complete set of equations, and inequalities, which is necessary for the statement of the design problem as a mathematical programming problem. This set of functions includes all equations needed to describe the geometry of the 4-bar linkage, the motion of its coupler guide, the objective function, and the design constraints.

3.1 Objective Function :

The object of the synthesis problem being considered here is to design a 4-bar linkage, where coupler guide follows as closely as possible some assigned positions, and whose crank rotations are as close as possible to a set of desired values. The desired positions are given in terms of coordinates of the coupler (x_i, y_i) point P and the orientation of the guide (ξ_i).

Since guide is fixed to the coupler

$$\overline{\Delta\eta}_i = \overline{\eta}_{i+1} - \overline{\eta}_i = \overline{\xi}_{i+1} - \overline{\xi}_i \qquad (i=1,2,\ldots.,n-1) \qquad (3.1)$$

The desired rotations of the crank are

$$\overline{\Delta\gamma}_i = \overline{\gamma}_{i+1} - \overline{\gamma}_i \qquad (i=1,2,\ldots.,n-1) \qquad (3.2)$$

Consider the 4-bar linkage shown in figure 4. Coordinates of coupler point are functions of linkage parameters $(X_1,\ldots,X_6,X_8,\ldots,X_{10})$ and of input angle $X_7(=\gamma)$. The 9 quantities $(X_1,\ldots,X_6,X_8,\ldots,X_{10})$ are taken as design variables in the synthesis problem. We define a design vector $\overline{D}$ as

$$D = \begin{bmatrix} X_1 \\ \vdots \\ X_6 \\ X_8 \\ \vdots \\ X_{10} \end{bmatrix} \tag{3.3}$$

Any set of values for 9 design variables, $\overline{D}$ is called a 'design', even if design is absurd. The coordinates of coupler point may be written as

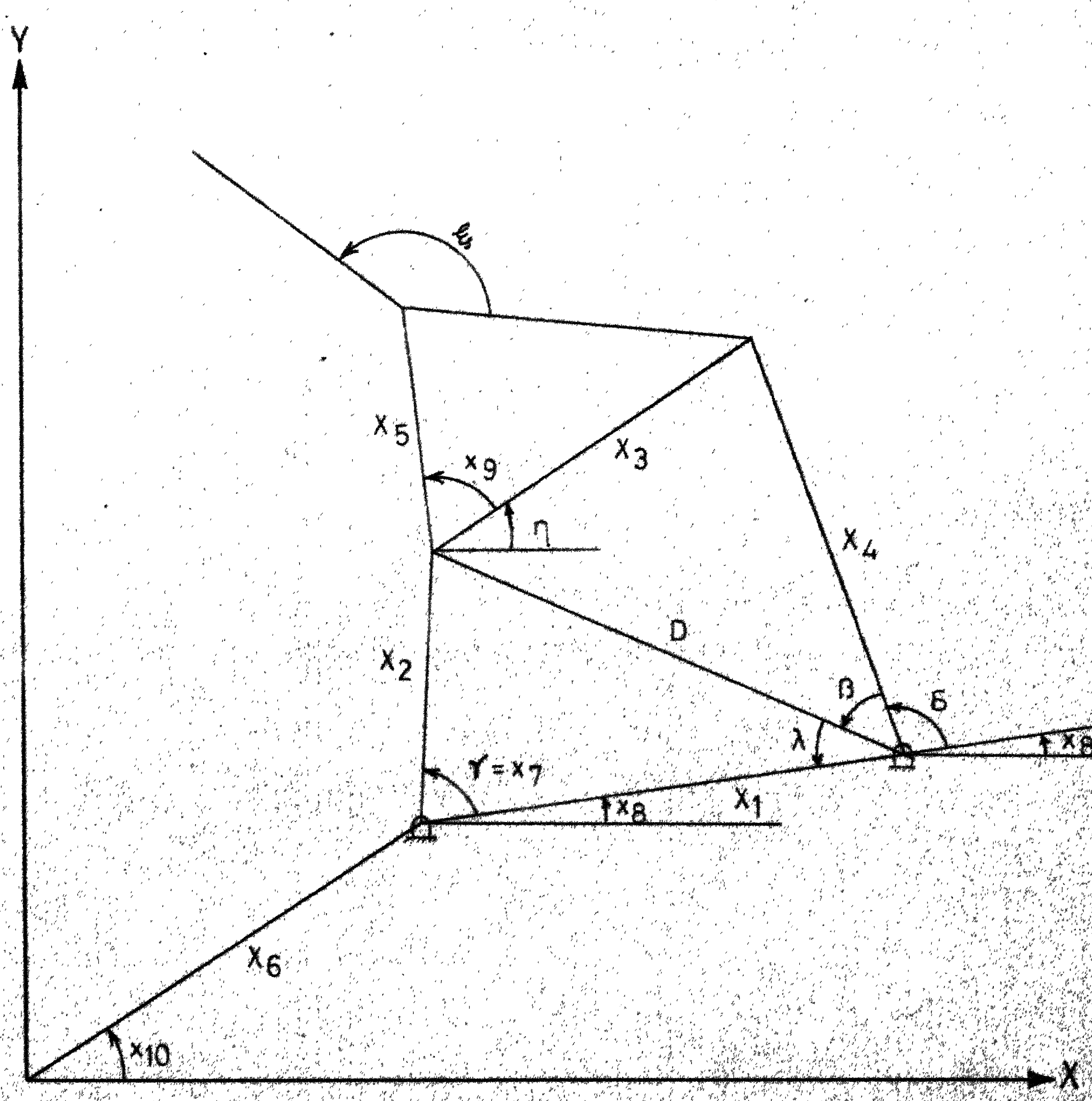
$$x_p = x\,(\overline{D}, \gamma_1, \gamma_2, \ldots, \gamma_n) \tag{3.4}$$

$$y_p = y\,(\overline{D}, \gamma_1, \gamma_2, \ldots, \gamma_n) \tag{3.5}$$

and orientation of the guide may be written as

$$\Delta\eta = \Delta\eta(\overline{D}, \gamma_1, \gamma_2, \ldots, \gamma_n) \tag{3.6}$$

An error function may be formulated as the sum of three functions, $E_c(\overline{D}, \gamma_1, \ldots, \gamma_n)$, $E_a(\overline{D}, \gamma_1, \ldots, \gamma_n)$ and $E_0(\overline{D}, \gamma_1, \ldots, \gamma_n)$, which define the coordinate matching error, crank angle matching error and guide orientation matching error.



TERMINOLOGY FOR FOUR- BAR LINKAGE IN OPTIMIZATION

FIG. 4

Starting with the initial design of a 4-bar linkage, the crank is rotated through given angles, each time solving the linkage for coordinates of coupler point and orientation of the guide in terms of the design vector $\bar{D}$ and X_7, and thus formulating two error functions instead of the three, as mentioned above. Thus, the total error function is

$$E(\bar{D}, X_7) = \sigma_1 E_c(\bar{D}, \gamma_1, \ldots, \gamma_n) + \sigma_2 E_0(\bar{D}, \gamma_1, \ldots, \gamma_n) \tag{3.7}$$

where, σ_1 and σ_2 are weighting factors chosen to give the desired relative importance of two types of errors.

Now including γ in design vector $\bar{D}$, we have

$$E(\bar{D}) = \sigma_1 E_c(\bar{D}) + \sigma_2 E_0(\bar{D}) \tag{3.8}$$

The 'best' design is the design which minimizes the total error, $E(\bar{D})$, and satisfies all the design constraints.

What now remains in the development of the objective function is to determine the coordinates of the coupler point and guide orientation as function of design vector $\bar{D}$. Referring to figure 4

$$D = (X_1^2 + X_2^2 - 2X_1X_2 \text{ Cos } (X_7))^{1/2} \tag{3.9}$$

$$\lambda = \text{Sin}^{-1} \left(\frac{X_2 \text{ Sin } (X_7)}{D}\right) \tag{3.10}$$

$$\beta = \text{Cos}^{-1} \left(\frac{D^2 + X_4^2 - X_3^2}{2DX_4}\right) \tag{3.11}$$

$$\delta = \pi - \beta - \lambda \tag{3.12}$$

$$X_{3x} = X_1 \text{ Cos } (X_8) + X_4 \text{ Cos } (\delta + X_8) - X_2 \text{ Cos } (X_7 + X_8) \tag{3.13}$$

$$X_{3y} = X_1 \, Sin \, (X_8) + X_4 \, Sin \, (\delta + X_8) - X_2 \, Sin \, (X_7 + X_8) \qquad (3.14)$$

$$\eta = \tan^{-1} \left(\frac{X_{3y}}{X_{3x}} \right)$$

The coordinates of coupler point can be written as

$$x_P = X_6 \, Cos \, (X_{10}) + X_2 \, Cos \, (X_7 + X_8) + X_5 \, Cos \, (X_9 + \eta) \qquad (3.15)$$

$$y_P = X_6 \, Sin \, (X_{10}) + X_2 \, Sin \, (X_7 + X_8) + X_5 \, Sin \, (X_9 + \eta) \qquad (3.16)$$

Now the total error function is

$$E(\bar{D}) = \sigma_1 \sum_{i=1}^{n} (x_P - x)^2 + (y_P - y)^2 + \sigma_2 \sum_{i=1}^{n-1} \left[(\eta_{i+1} - \eta_1) - \bar{\Delta\eta}_i \right]^2 \qquad (3.17)$$

3.2 Design Constraints :

The design constraints, a satisfaction of which distinguishes acceptable designs from unacceptable designs can be grouped into two categories. Any constraint which restricts the range of design variables for reasons other than the direct consideration of performance of the design is called a side constraint. A constraint which is derived from the explicit consideration of the performance of the design is called a behaviour constraint.

3.2.1 Realizability Constraint :

We may say that all link lengths should be greater than a given minimum values.

$$X_1 - X_{min} \geq 0 \tag{3.18}$$

$$X_2 - X_{min} \geq 0 \tag{3.19}$$

$$X_3 - X_{min} \geq 0 \tag{3.20}$$

$$X_4 - X_{min} \geq 0 \tag{3.21}$$

Also requiring that X_7 should be greater than zero and less than 360 degrees give

$$X_7 \geq 0 \tag{3.22}$$

$$2\pi - X_7 \geq 0 \tag{3.23}$$

3.2.2 Crank Rocker Constraint :

Requiring that the 4-bar linkage be a crank-rocker mechanism adds another set of five constraints. The Grashoff Inequality [19] states that if the sum of the lengths of longest and shortest link is less than the sum of other two links, and if the shortest link is the input link, then the 4-bar linkage will be a crank rocker mechanism. This gives

$$X_1 - X_2 \geq 0 \tag{3.24}$$

$$X_3 - X_2 \geq 0 \tag{3.25}$$

$$X_4 - X_2 \geq 0 \tag{3.26}$$

$$X_3 + X_4 - X_1 - X_2 \geq 0 \tag{3.27}$$

and $$(X_1 - X_2)^2 - (X_3 - X_4)^2 \geq 0 \tag{3.28}$$

3.2.3 Compactness Constraint :

Since there are usually limitations on the span in which the mechanism is to be used, a set of constraints is

used to limit the length of each link to less than a prescribed maximum length. It is not necessary to place size limit on link X_1 because the length of X_1 is limited by constraints placed on the location of the pivot points O_A and O_B.

$$X_{max} - X_2 \geqslant 0 \qquad (3.29)$$

$$X_{max} - X_3 \geqslant 0 \qquad (3.30)$$

$$X_{max} - X_4 \geqslant 0 \qquad (3.31)$$

3.2.4 Pivot location Constraints :

In some mechanism design problems, it may be necessary to limit the pivot points O_A and O_B (figure 5) to lie in prescribed regions of the XY plane. For simplicity a circular area is chosen here, however it could be taken to be of any definable portion of the plane.

The limitation of the location of O_A gives

$$|\bar{X}_A - \bar{X}_6| \leqslant R_A \qquad (3.32)$$

or in terms of design variables

$$R_A^2 - X_A^2 - X_6^2 + 2X_A X_6 \text{ Cos } (\measuredangle X_A - X_{10}) \geqslant 0 \qquad (3.33)$$

and the limitation of the location of O_B

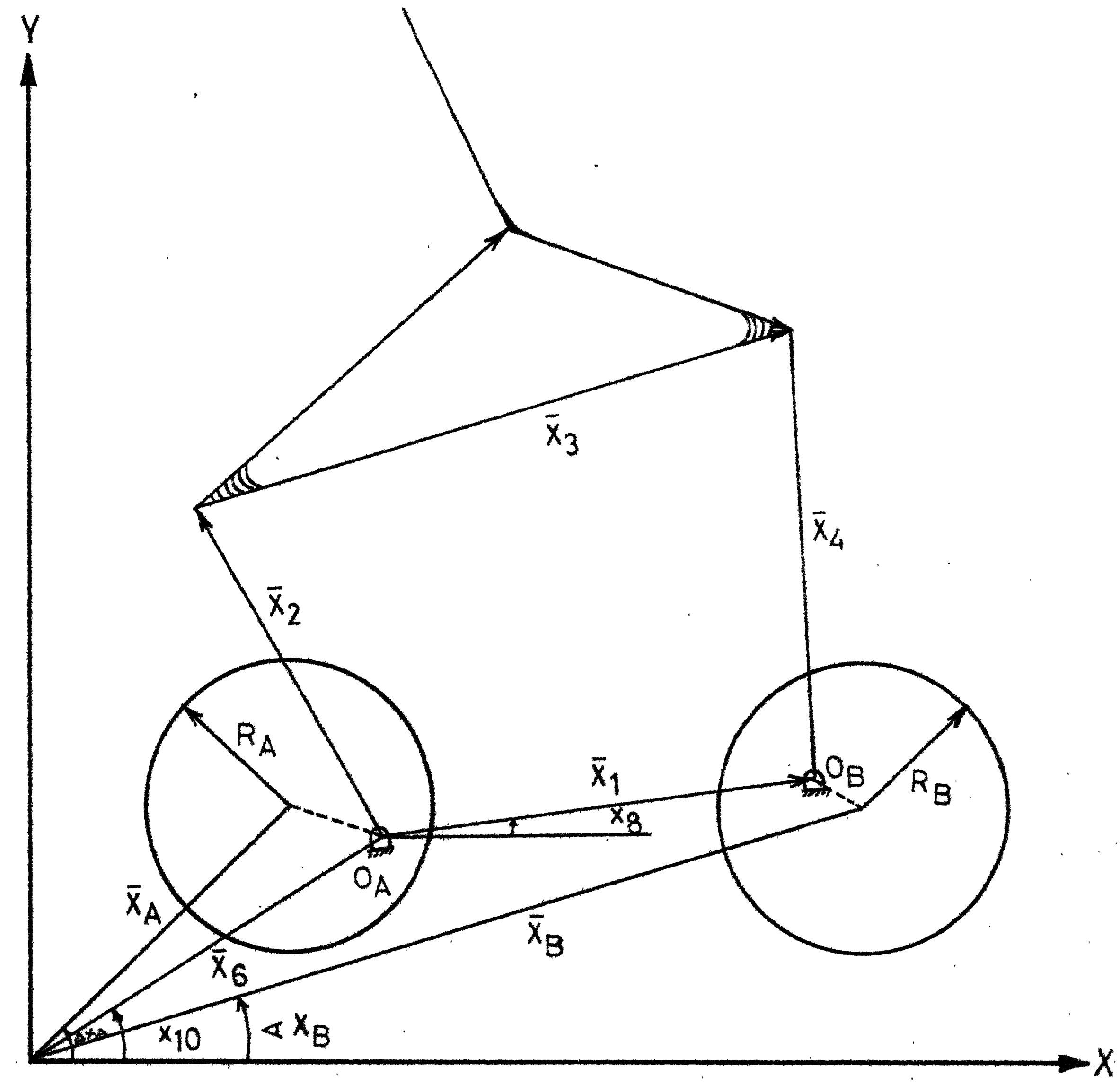
$$|\bar{X}_B - \bar{X}_6 - \bar{X}_1| \leqslant R_B \qquad (3.34)$$

But

$$\bar{X}_B - \bar{X}_6 - \bar{X}_1{}^2 = (\bar{X}_B - \bar{X}_6 - \bar{X}_1) \cdot (\bar{X}_B - \bar{X}_6 - \bar{X}_1)$$

$$= X_6^2 + X_1^2 + X_B^2 + 2X_1 X_6 \text{ Cos } (X_{10} - X_8)$$

$$-2X_B X_6 \text{Cos}(\measuredangle X_B - X_{10}) - 2X_B X_1 \text{Cos } (\measuredangle X_B - X_8) \qquad (3.35)$$



RESTRICTION ON THE LOCATION OF THE PIVOT POINT

FIGURE 5

Therefore the inequality (3.42) may be written as

$$R_B^2 - X_B^2 - X_6^2 - X_1^2 - 2X_1X_6\cos(X_{10}-X_8) + 2X_BX_6\cos(\lambda_B X_B - X_{10})$$
$$+ 2X_BX_1\cos(\lambda_B X_B - X_8) > 0 \qquad (3.36)$$

where R_A, R_B, X_A, $\lambda_A X_A$, X_B, $\lambda_B X_B$ can be chosen by the designer.

3.2.5 Force Moment Constraints :

There is limitation of the torque and forces in the links as a result of the force applied on the coupler point.

The axial reaction force $\overline{K}_4$ in link 4, $\overline{K}_2$ in link 2 and the torque $\overline{T}_2$ in the input link can be expressed as Appendix B.

$$\overline{K}_4 = \frac{X_5\, W(x)\, \sin(\eta + X_9 - \theta(x))}{X_3\, \sin(X_8 + \delta - \eta)} \qquad (3.37)$$

$$\overline{K}_2 = -\overline{K}_4 \cos(\delta - X_7) - W(x)\cos(\theta(x) - X_7 - X_8) \qquad (3.38)$$

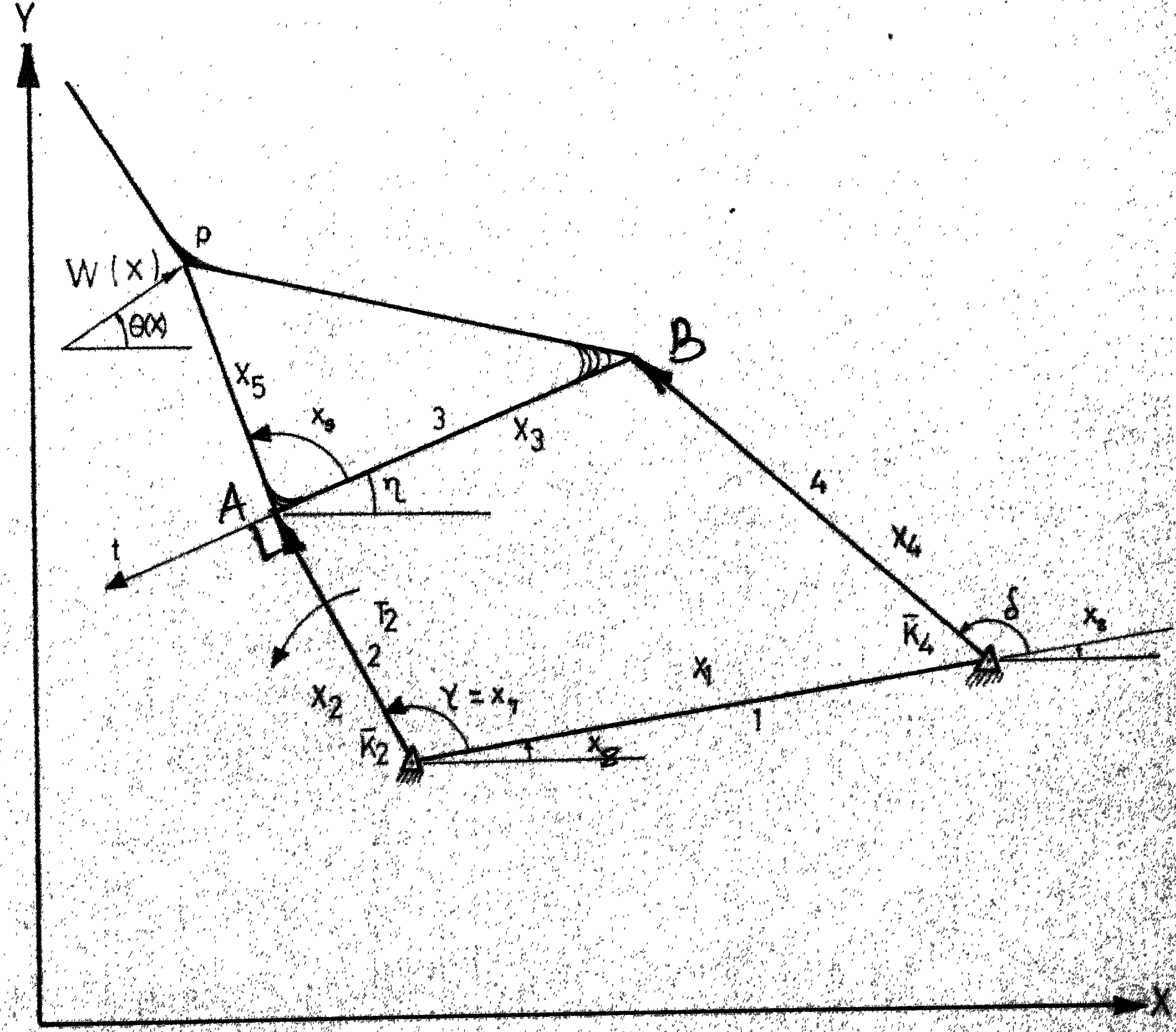
and

$$\overline{T}_2 = -X_2\, K_4 \sin(\delta - X_7) - X_2 W(x) \sin(\theta(x) - X_7 - X_8) \qquad (3.39)$$

If K_2, K_4 and T_2 are maximum allowable values of $\overline{K}_2$, $\overline{K}_4$ and $\overline{T}_2$, then

$$K_4^2 - \left[\frac{X_5\, W_i\, \sin(\eta_i + X_{9i} - \theta_i)}{X_3\, \sin(X_{8i} + \delta_i - \eta_i)}\right]^2 > 0 \qquad (3.40)$$

$$K_2^2 - \left[W_i \cos(\theta_i - X_{7i} - X_{8i}) + \frac{X_5\, W_i \sin(\eta_i + X_{9i} - \theta_i)}{X_3 \sin(X_{8i} + \delta_i - \eta_i)}\right.$$
$$\left.\cos(\delta_i - X_{7i})\right]^2 > 0 \qquad (3.41)$$



FREE BODY DIAGRAM OF COUPLER LINK

FIGURE 6

$$T_2^{\,2} - \left[X_2 W_i \mathrm{Sin}(\theta_i - X_{8i}) + X_2 \frac{X_5 W_i \mathrm{Sin}(\eta_i + X_{9i} - \theta_i)}{X_3 \mathrm{Sin}(X_{8i} - \delta_i - \eta_i)} \mathrm{Sin}(\delta_i - X_{7i})\right]^2 \geq 0 \tag{3.42}$$

3.2.6 Transmission Angle Constraints :

Transmission angle should be as close to $\pi/2$ as possible for better transmission characteristics. Maximum and minimum transmission angles occur when crank is in line of base link.

$$\mu_{min} = \mathrm{Cos}^{-1} \frac{X_3^{\,2} + X_4^{\,2} - (X_1 - X_2)^2}{2X_3\, X_4} \tag{3.43}$$

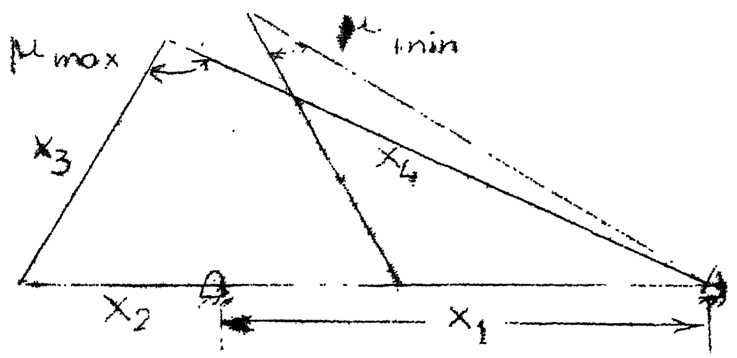
$$\mu_{max} = \mathrm{Cos}^{-1} \frac{X_3^{\,2} + X_4^{\,2} - (X_1 + X_2)^2}{2X_3\, X_4} \tag{3.44}$$

One common practice is that the deviation of μ_{max} and μ_{min} should be less than 30°.

$$\frac{\pi}{6} - \left|\mu_{max} - \frac{\pi}{2}\right| \geq 0 \tag{3.45}$$

and

$$\frac{\pi}{6} - \left|\frac{\pi}{2} - \mu_{min}\right| \geq 0 \tag{3.46}$$



CHAPTER IV

SOLUTION ALGORITHM

The method employed in the solution of constrained minimization problem is discussed in this Chapter. The constrained problem is first converted into an unconstrained minimization problem and then solved as a sequence of unconstrained problems. Each of the unconstrained minimization problems in the sequence is solved by the iterative descent method in which the gradient vector is approximated by differences.

4.1 Conversion to Unconstrained Minimization :

The general mathematical programming problem is to determine a vector $\bar{X}$ that minimizes the function $f(\bar{X})$ subject to the constraints $g_i(\bar{X}) \geq 0$, $(i = 1,2,\ldots,m)$.

Fiacco and McCormick [20,21] have developed an alogorithm for transforming this mathematical programming problem with constraints into a sequence of unconstrained minimization problems. The procedure is based on the minimization of a new function

$$p(\bar{X},r) = f(\bar{X}) + r \sum_{i=1}^{m} \frac{1}{g_i(\bar{X})} \qquad (4.1)$$

over a strictly monotonic decreasing sequence of r-values $\{r_K\}$.

Choose $r_1 > 0$ and a point $\bar{X}_0$ which lies strictly within the constraint set, i.e. $g_i(\bar{X}_0) > 0$, $(i = 1,2,\ldots,m)$.

It is expected that a minimum of p $(\overline{X},r_1)$ exists inside the constraint set, since on the boundary of the constraint set some of $g_i(\overline{X})$ must equal to zero, and $p(\overline{X},r_1) \to \infty$. The minimum point of $p(\overline{X},r_1)$ depends on the value of r_1, and is denoted by $\overline{X}(r_1)$. The point $\overline{X}(r_1)$ lies inside the constraint set. By reducing the value of r, the influence of the summation term, which penalizes closeness to the constraint boundaries, is reduced, and, in minimizing $p(\overline{X},r)$ more emphasis is placed upon decreasing $f(\overline{X})$ If the minimization process is repeated for a decreasing sequence of r values, $r_1 > r_2 > \cdots\cdot r_K > 0$, each minimum point $\overline{X}(r_K)$ can move closer to the boundary of the constraint set if it is profitable in terms of reducing $f(\overline{X})$. This method is particularly attractive for problems, such as the 4-bar linkage problem, which have nonlinear constraints, since it approaches the solution from points inside the constraint set, and thus, avoids the difficult movement along the nonlinear boundary of the set, which is required in other methods.

Fiacco and McCormick have shown that if

(i) the interior of the constraint set is nonempty,

(ii) the functions $f(\overline{X})$ and $g_i(\overline{X})$, $(i=1,\ldots\ldots,m)$ are twice continuously differentiable,

(iii) the set of points in the constraint set for which $f(\overline{X}) \leq v_0$ is bounded for every finite v_0, and

(iv) the function $f(\overline{X})$ is bounded below for $\overline{X}$ in the constraint set;

then, the optimal solution to the unconstrained problem approaches a local minimum of the constrained problem as the value of r approaches zero. If, in addition

(v) $f(\bar{X})$ and $g_i(\bar{X})$, $(i=1,2,\ldots,m)$ are convex functions, and

(vi) $p(\bar{X},r)$ is strictly convex in the interior of the constraint set for every $r > 0$,

then, the optimal solution to the unconstrained problem approaches the absolute minimum of the constrained problem as r approaches zero.

In order to apply the Fiacco McCormick algorithm to the 4-bar linkage problem, the constraints must be expressed as penalty functions. The penalty function for the constraints in section (3.21) to (3.26), are simply the reciprocal of the constraint functions, since these are all discrete constraints.

4.2 Choice of Initial r :

To start the algorithm a decision has to be taken about the initial value of r and factor c, the factor by which the value of r shall be decreased. The algorithm, imposes no restriction on the value of r anc C except that $r > 0$ and $C > 1$. With large r-values the number of computations required to reach the constrained minimum increases tremendously while with small value of r, the function is highly distorted and requires initial solution very close to actual minima.

4.3 Method of Unconstrained Minimization :

An unconstrained minimization of the function $p(\overline{D},r)$ must be carried out over the design vector, $\overline{D}$, for each value of r used in the Fiacco-McCormick algorithm. The method of unconstrained minimization used in this dissertation is a powerful quadratically convergent iterative descent method, orginally given by Davidon [22], and subsequently modified by Fletcher and Powell [23, 24].

The method is based on the properties of the quadratic function

$$\emptyset(\overline{X}) = \emptyset_o + \overline{X}^T\overline{b} + \tfrac{1}{2}\,\overline{X}^T A\overline{X} \qquad (4.2)$$

where $\emptyset_o$ is ascalar, $\overline{X}^T = (\xi_1, \xi_2, \ldots\ldots \xi_n)$, $\overline{b}$ is a constant n-vector, and A is a symmetric, positive definite matrix. The gradient of the function is

$$g = \overline{b} + A\overline{X} \qquad (4.3)$$

If we consider the quadratic form (4.2), then, given the matrix $A_{ij} = \dfrac{\partial^2 \emptyset}{\partial X_i \partial X_j}$, we can calculate the displacement between the point $\overline{X}$ and the minimum $\overline{X}_o$ as

$$\overline{X}_o - \overline{X} = -A^{-1}\overline{g} \qquad (4.4)$$

In this method A^{-1} is not evaluated directly. Instead a matrix H is used which may initially (H_o) be chosen to be any positive definite symmetric matrix (identify matrix I). The method is descent method which proceeds towards the minimum of the function by a sequence of linear minimizations. The directions along which the function is

minimized are chosen so that if the function being minimized is quadratic function (4.2), its minimum will be found after n linear minimization.

A typical iteration in the minimization of a function $f(\bar{X})$ proceeds as follows :

1. Given the starting point $\bar{X}_i$ and the gradient of the function evaluated at $\bar{X}_i$, $g_i = f(\bar{X}_i)$, the direction along which the (i+1) minimum will be sought is calculated from

$$\bar{d}_i = -\left[H_i\right] \bar{g}_i \tag{4.5}$$

where H is positive definite matrix.

2. Find σ_i so that $f(\bar{X}_i + \sigma_i \bar{d}_i)$ is a minimum along the line $\bar{X}_i + \sigma \bar{d}_i$

3. Set $\bar{X}_{i+1} = \bar{X}_i + \sigma_i \bar{d}_i$

4. Calculate the new gradient vector $\bar{g}_{i+1} = \nabla f(\bar{X}_{i+1})$ and set $\bar{y}_i = \bar{g}_{i+1} - \bar{g}_i$

5. Calculate the new H matrix H_{i+1} by

$$\left[H_{i+1}\right] = \left[H_i\right] + \sigma_i \frac{\bar{d}_i \bar{d}_i^T}{\bar{d}_i^T \bar{y}_i} - \frac{\left[H_i\right] \bar{y}_i \bar{y}_i^T \left[H_i\right]}{\bar{y}_i^T H_i \bar{y}_i} \tag{4.6}$$

6. Begin the next iteration from step 1.

The modification of the matrix H_i (step 5) is done to take into account local characteristics of the function in order to avoid the zig-zag characteristics common to many other minimization techniques, such as the

method of steepest descent. It can be shown that the matrix $[H_i]$ approximates the inverse of the matrix of second partial derivatives of $f(\overline{X})$ evaluated at the minimum because as the minimum is approached the second order terms predominate in Taylor's expansion. It can also be shown that if the function being minimized is the quadratic function (4.2), then

$$[H_{n-1}] = [A]^{-1}$$

In case minimum along a line is not found, a new $[H]$ matrix is not calculated in step 5. The new direction is calculated from equation (4.5), using the new gradient, g_{i+1}, and the old $[H]$ matrix. This always helps to ensure that $[H]$ is always positive definite, and thus the directions $\overline{d}_i$ will always be directions of descent. It is found that $[H]$ is no longer positive definite, then H_i is reset to the identity matrix, $[I]$, and the iteration is continued.

The unconstrained minimization procedure is terminated when the components of both direction $\overline{d}_i$ and the actual step $\tau_i\ \overline{d}_i$ become less than a prescribed level.

4.4 Gradient :

In order to calculate the directions used in Fletcher and Powell's method, it is necessary to calculate the gradient vector of the function $f(\overline{X})$ during the course of each iteration. Since analytic expressions for the

partial derivatives of $p(\overline{X}\ r)$ are quite cumbersome, the gradient vector is approximated by differences.

Let $f = f(\xi_1, \ldots\ldots, \xi_j \ldots\ldots \xi_n)$ (4.7)

and $f^* = f^*(\xi_1, \ldots\ldots, \xi_j + \Delta\xi_j \ldots \xi_n)$ (4.8)

Then, by Taylor's series expansion and neglecting higher order terms

$$\frac{\partial f}{\partial \xi_j} = \frac{f^* - f}{\Delta \xi_j} \tag{4.9}$$

A simple scheme is followed to calculate $\Delta\xi_j$. $\Delta\xi_j$ can be approximated as $\epsilon\,\xi_j$, where ϵ is a small positive quantity. Each time when f is changed to f^*, all constraints are checked to ensure that none is violated. If not, $\Delta\xi_j$ should be replaced by $\Delta\xi_j$. Even if all constraints are not satisfied $\Delta\xi_j$ should be replaced by $\epsilon^2 X_i$. Thus all components of the gradient are calculated. If any ξ_j turns out to be zero, corresponding $\Delta\xi_j$ should be replaced by a small positive quantity ϵ.

4.5 Linear Minimization :

In the Fletcher and Powell procedure it is necessary to find the value of $\alpha = \alpha_i$ such that $f(X_i + \alpha_i \overline{d}_i)$ is the minimum of $f(\overline{X})$ along the line $\overline{X} = \overline{X}_i + \alpha\,\overline{d}_i$. The method used for this minimization along a line is Golden Section Search[25]. Since the Fletcher-Powell technique is based on the assumption that $f(\overline{X})$ can be approximated by a quadratic function in the vicinity of the minimum,

the linear minimization scheme is based on the minimization of a quadratic function.

If $\overline{X}_i$ and $\overline{d}_i$ are considered fixed, $f(\overline{X})$ can be considered as a function of σ only.

$$f(\overline{X}) = f(\overline{X}_i + \sigma \overline{d}_i) = f(\sigma) \qquad (4.10)$$

Then, σ_i is the value of σ for which $f(\sigma)$ is minimum. Let α_{mid} be the value of α beyond which constraints are violated.

First we start with some positive value of α and check whether replacing $\overline{X}_i$ by $\overline{X}_i + \alpha \overline{d}_i$ all constraints are satisfied or not. Thus we determine the bounds of α as α_{min} for which constraints are satisfied and α_{max} for which constraints are not satisfied.

$f(\alpha)$ is assumed as

$$f(\alpha) = C_1 \left[1 + \frac{(\alpha - \alpha_{min})^2}{(\alpha_{max} - \alpha_{min})^2} \right] \quad \text{for } \alpha < \alpha_{mid} \qquad (4.11)$$

where C_1 is a constant

$$f(\alpha) = C_2 \qquad \text{for } \alpha > \alpha_{mid}$$

C_2 is a small number.

By using Golden Section Search we can find the maximum $f(\alpha)$ as $f(\alpha_{mid})$. After determining α_{mid}, Golden Section search can again be used to find the minimum of $f(\alpha)$ as $f(\alpha_i)$ between $\alpha = 0$ and $\alpha = \alpha_{mid}$.

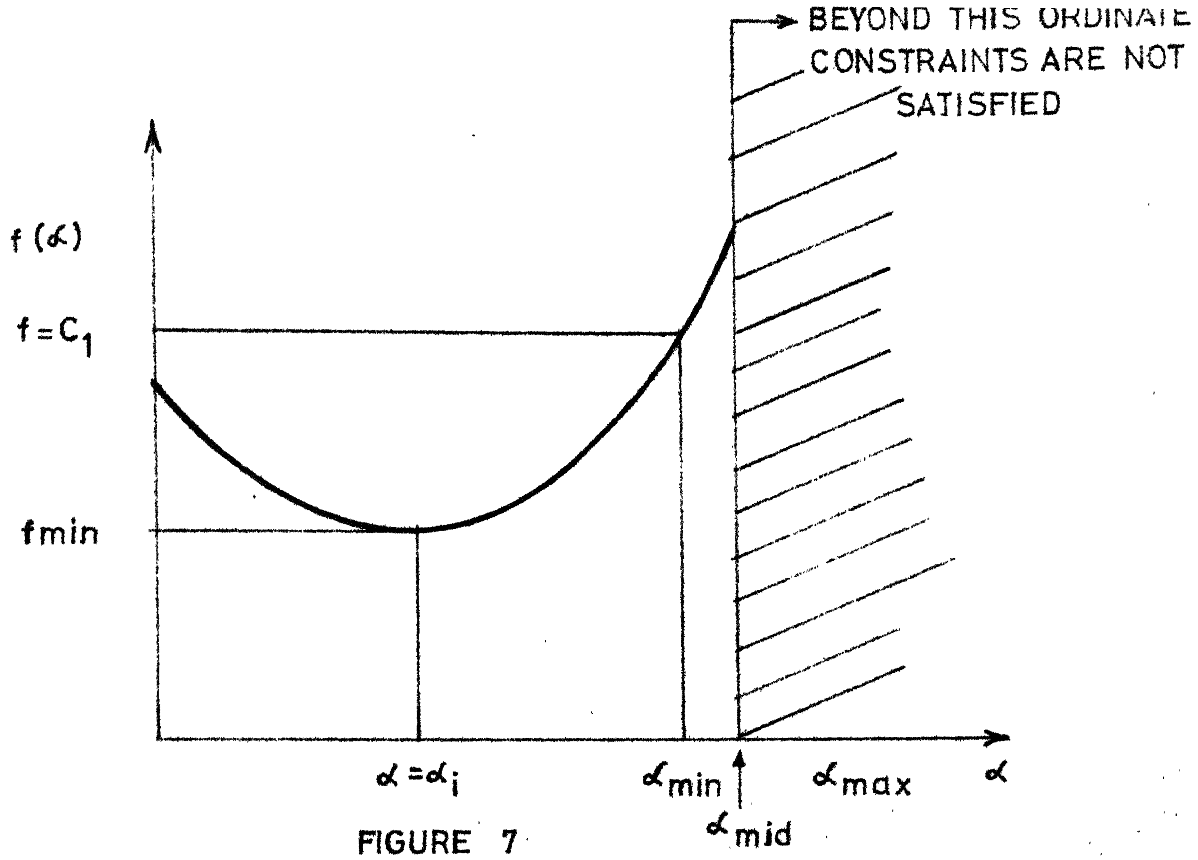


FIGURE 7

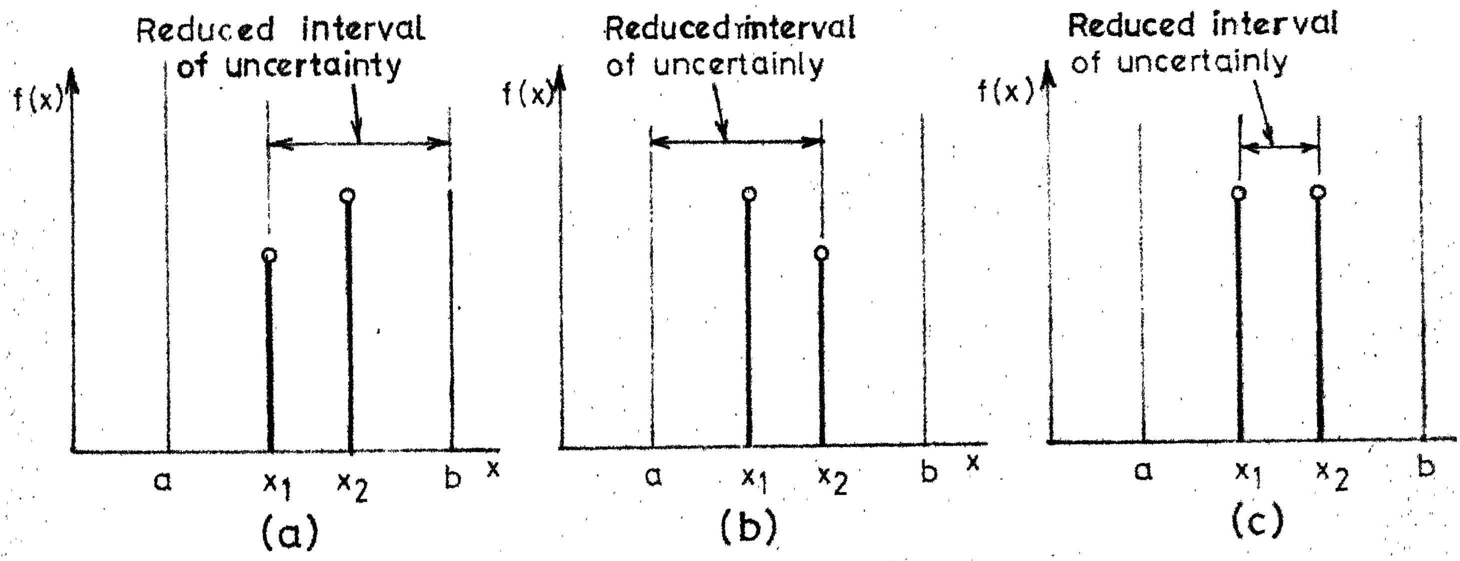


FIGURE 8

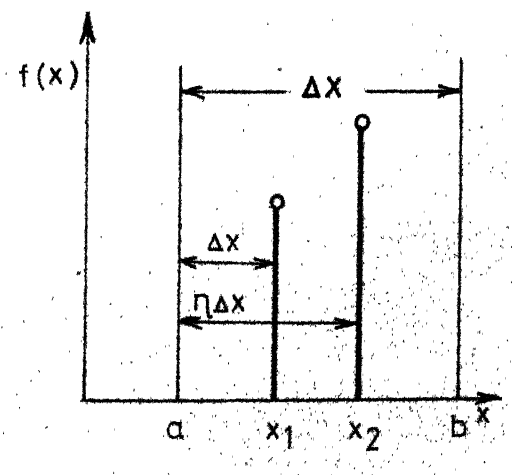


FIGURE 9

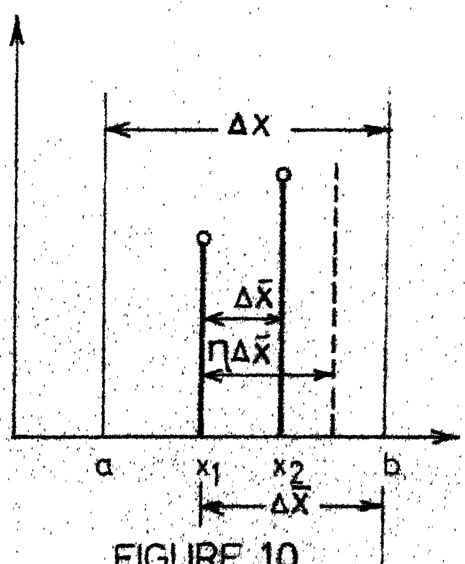


FIGURE 10

Golden Section Search proceeds as follows :

Considering the unimodal function $f(X)$ in the interval $a \leqslant X \leqslant b$ evaluated at two intermediate ordinates as shown in figure 8. In the case shown in figure (8.a), the extremum lies in the interval $X_1 \leqslant X \leqslant b$. In the case shown in figure (8b) the extremum lies in the interval $a \leqslant X \leqslant X_2$ and in $X_1 \leqslant X \leqslant X_2$ for case figure (8c).

In this procedure ξ and η are so defined that if the region $a \leqslant X \leqslant X_1$ is eliminated, the ordinate X_2 is in the proper location; that it is the first ordinate of the new interval, $\Delta\overline{X} = b - X_1$ i.e. located at $\xi \Delta \overline{X}$ from X_1, the new left end of interval of uncertainty, figure 10. Similarly, if the region $X_2 \leqslant X \leqslant b$ was eliminated by X_1 being greater than X_2, then X_1 would be in the proper location to be the second ordinate in the interval $\Delta \overline{X}^1 = (X_2 - a)$ i.e. located at $\eta\ \overline{X}^1$ from a.

This readiness for either alternative demands a symmetry in the location of X_1 and X_2 which gives

$$\xi = 0.381966011$$

$$\eta = 0.618033989.$$

CHAPTER V

NUMERICAL EXAMPLES

In this Chapter few numerical examples have been solved.

5.1 Three-Position Synthesis :

x_{pi}	10.9, 8.4, 6.0		
y_{pi}	11.7, 13.0, 5.2		
t_1	40.0 deg.	u_1	-19.0 deg.
t_2	80.0 deg.	u_2	-20.0 deg.

Arbitrary Parameter

v_1	5.0 deg.
v_2	25.0 deg.

Solution

X_1	32.8
X_2	4.7
X_3	15.2
X_4	24.1
X_5	5.5
X_6	14.1
γ	60.4 deg.
$\measuredangle X$	6.2 deg.
$\measuredangle X_5 - \measuredangle X_3$	97.0 deg.
$\measuredangle X_6$	18.6 deg.

5.2 Four-Position Synthesis :

Desired Motion

x_{pi}	10.9, 9.6, 7.3, 5.6
y_{pi}	11.7, 12.8, 11.9, 9.7
u_i	-12.0, -19.0, -20.0 deg.

Arbitrary Parameters

t_1	40.0 deg.
v_1	7.0 deg.

Solution

X_1	32.0
X_2	5.4
X_3	2.6
X_4	34.7
X_5	19.3
X_6	32.5
γ	205.0 deg.
$\measuredangle X_1$	147.4 deg.
$\measuredangle X_5 - \measuredangle X_1$	-18.4 deg.
$\measuredangle X_6$	94.7 deg.
v_2	7.2 deg.
v_3	3.1 deg.
t_2	38.1 deg.
t_3	11.3 deg.

5.3 Five-Position Synthesis :

Desired Motion

x_{pi}	10.9, 9.6, 7.3, 5.3, 5.6
y_{pi}	11.7, 12.8, 11.9, 8.3, 6.0
u_i	-12.0, -19.0, -19.0, -7.0 deg.

Solution

X_1	12.0
X_2	4.0
X_3	9.5
X_4	8.0
X_5	4.0
X_6	8.5
γ	31.0 deg.
$\measuredangle X_1$	11.0 deg.
$\measuredangle X_5 - \measuredangle X_3$	29.0 deg.
$\measuredangle X_6$	36.0 deg.

5.4 Seven-Position Synthesis - Optimization :

Note : All angles are in degrees.

Desired Motion

x_{pi}	10.9, 9.6, 7.3, 5.6, 5.3, 5.3, 5.6
y_{pi}	11.7, 12.8, 11.9, 9.7, 8.3, 7.1, 6.0
$\overline{\Delta\eta}_i$	-12.0, -7.0, -1.0, 1.0, 6.0, 5.0
$\overline{\Delta\gamma}_i$	40.0, 40.0, 40.0, 40.0, 40.0, 40.0

Constraints

1. Crank-rocker mechanism
2. $X_1, X_2, \ldots\ldots, X_6 \geq 0$
3. $X_{max} = 20.0$; $X_{min} = 1.0$
4. W_i = 10.0, 10.0, 10.0, 10.0, 10.0, 10.0, 10.0
5. θ_i = 10.0, 10.0, 10.0, 10.0, 10.0, 10.0, 10.0
6. $R_A = 4.0$, $X_A = 8.0$ and $\angle X_A = 30.0$
   $R_B = 4.0$, $X_B = 20.0$ and $\angle X_B = 25.0$
7. $T_2 = 50.0$, $K_2 = 20.0$ and $K_4 = 30.0$

Solution

1. Value of C = 4.0

Design Variables	Initial Solution	Final Solution
X_1	12.0	11.9492
X_2	4.0	3.9783
X_3	9.5	9.5015
X_4	8.0	7.9859
X_5	4.0	4.0320
X_6	8.5	8.5015
γ	31.0	24.8317
γX_1	11.0	13.1213
$\gamma X_5 - \gamma X_3$	29.0	19.1214
γX_6	36.0	34.8319
r	10.0	.0015
$E(\bar{D})$	8.2061	1.0664
$p(\bar{D}, r)$	123.5217	1.0664

CHAPTER VI

CONCLUSIONS AND DISCUSSIONS

In the analytic approach to the problem it is found that 4-position problem will find wide spread usage, as the designer has the control over arbitrary parameters and can obtain variety of mechanisms. Though the 5-position problem will give more accurate results for a given N-position problem, but, it is complicated one. In this case we have to solve a polynomial of 8th order which is really a cumbersome job.

In optimized solution it is known that conditions (i)-(iv) (Section 4.1) are satisfied for the 4-bar linkage problem. However, since there is no simple way to test the convexity conditions, (v) and (vi), for this nonlinear problem, it is not known whether the optimal solution to the unconstrained minimization problem is the optimal solution to the constrained minimization problem or only a local minimum of the constrained problem. It is expected that the method will tend to converge to a minimum which has the greatest influence in the vicinity of the starting point. Therefore, the final design for a given 4-bar linkage problem solved by the method presented here is dependent upon the choice of initial design.

The main problem which arises in using the Penalty Function Approach is the choice of initial value of 'r'. Fiacco and McCormick have presented two methods for choosing the initial value of 'r', both of which depend on some knowledge of the characteristics of the gradient vector. In using the finite difference formulae to compute the gradient at each step, these necessary factors are not available. In the present work initial 'r' is taken to be '10' which gives pretty good results. Two methods of finding initial 'r' are given in Appendix C.

The gradient calculation which has been done in the present work is not very accurate. A better scheme is given by G.W. Stewart, III[(1)] for estimating the gradient vector by difference.

Other areas of further study include investigation of the present problem with design constraints such as excluded region for coupler motion, dynamic characteristics and material property.

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(1) G.W.Stewart, III, "A Modification of Davidon's Minimization Method to Accept Difference Approximations of Derivatives", Journal of the Association for Computing Machinery, Vol.14, No.1, January 1967, pp 72-83.

APPENDIX A

SYNTHESIS OF FIVE POSITION PROBLEM

We can write position equation for this case as

Loop 1 :

$$Z_1 + Z_4 - Z_3 = Z_2 \tag{A.1}$$

$$Z_1 + Z_4e^{jv_1} - Z_3e^{ju_1} = Z_2e^{jt_1} \tag{A.2}$$

$$Z_1 + Z_4e^{jv_2} - Z_3e^{ju_2} = Z_2e^{jt_2} \tag{A.3}$$

$$Z_1 + Z_4e^{jv_3} - Z_3e^{ju_3} = Z_2e^{jt_3} \tag{A.4}$$

$$Z_1 + Z_4e^{jv_4} - Z_3e^{ju_4} = Z_2e^{jt_4} \tag{A.5}$$

Loop 2 :

$$Z_6 + Z_2 + Z_5 = r_0e^{jw_0} \tag{A.6}$$

$$Z_6 + Z_2e^{jt_1} + Z_5e^{jv_1} = r_1e^{jw_1} \tag{A.7}$$

$$Z_6 + Z_2e^{jt_2} + Z_5e^{jv_2} = r_2e^{jw_2} \tag{A.8}$$

$$Z_6 + Z_2e^{jt_3} + Z_5e^{jv_3} = r_3e^{jw_3} \tag{A.9}$$

$$Z_6 + Z_2e^{jt_4} + Z_5e^{jv_4} = r_4e^{jw_4} \tag{A.10}$$

It is clear that there are 20 unknowns, 8 for angles $t_1,\ldots,t_4$ and $v_1,\ldots,v_4$ and 12 for 6 vectors. There

are 20 real equations at our disposal. Hence a nontrivial solution can be obtained uniquely without assigning any parameter arbitrary.

Proceeding similar to section 2.7, we have

$$a_i Z_2 + b_i Z_5 = C_i \quad (i = 1,\ldots,4) \tag{A.11}$$

where

$$\left.\begin{aligned} a_i &= 1 - e^{jt_i} && \text{(A.12)} \\ b_i &= 1 - e^{ju_i} && \text{(A.13)} \\ c_i &= r_0 e^{jw_0} - r_i e^{jw_i} && \text{(A.14)} \end{aligned}\right\} \quad (i = 1,\ldots\ldots,4)$$

From set of equations (A.11) we get

$$b_i' Z_5 = c_i' \qquad (i = 2,\ldots\ldots,4) \tag{A.15}$$

where

$$\left.\begin{aligned} b_i' &= b_1 - \frac{b_i}{a_i} a_1 \\ c_i' &= c_1 - \frac{c_i}{a_i} a_1 \end{aligned}\right\} \quad (i = 2,\ldots\ldots,4) \tag{A.16}$$

Equations in (A.15) give the following two independent equations :

$$\frac{c_2'}{b_2'} = \frac{c_3'}{b_3'} \tag{A.17}$$

and

$$\frac{c_2'}{b_2'} = \frac{c_4'}{b_4'} \tag{A.18}$$

Expanding equation (A.17) in terms of e^{jt_i} and using following identities

$$d_1 = c_3 b_2 - c_2 b_3 \quad (A.19)$$

$$d_2 = c_1 b_3 - c_3 b_1 \quad (A.20)$$

$$d_3 = c_2 b_1 - c_1 b_2 \quad (A.21)$$

$$d_4 = -(d_1 + d_2 + d_3) \quad (A.22)$$

We get

$$d_1 \, e^{jt_1} + d_2 \, e^{jt_2} + d_4 = - d_3 \, e^{jt_3} \quad (A.23)$$

Multiplying by its complex conjugate

$$\bar{d}_1 \bar{e}^{jt_1} + \bar{d}_2 \bar{e}^{jt_2} + \bar{d}_4 = - \bar{d}_3 \bar{e}^{jt_3} \quad (A.24)$$

we get

$$\bar{d}_1 \, d_2 \, e^{j(t_2-t_1)} + d_1 \bar{d}_2 \, \bar{e}^{j(t_2-t_1)} + \bar{d}_4 d_1 e^{jt_1} + \bar{d}_1 d_4 \bar{e}^{jt_1} + \bar{d}_4 d_2 \, e^{jt_2} + d_4 \bar{d}_2 \bar{e}^{jt_2} = d_3 \bar{d}_3 \quad (A.25)$$

Defining

$$p = \bar{d}_1 d_2 \quad (A.26)$$

$$q = \bar{d}_4 d_1 \quad (A.27)$$

$$s = \bar{d}_4 d_2 \quad (A.28)$$

$$r = d_5/2 \quad (A.29)$$

Equation (A.25) can be written as

$$p\, e^{j(t_2-t_1)} + \bar{p}\, e^{-j(t_2-t_1)} + q\, e^{jt_1} + \bar{q}\, e^{-jt_1} + s\, e^{jt_2} + \bar{s}\, e^{-jt_2} + 2r = 0 \qquad (A.30)$$

Similarly equation (A.18) can be written as

$$p'\, e^{j(t_2-t_1)} + \bar{p}'\, e^{-j(t_2-t_1)} + q'\, e^{jt_1} + \bar{q}'\, e^{-jt_1} + s'\, e^{jt_2} + \bar{s}'\, e^{-jt_2} + 2r' = 0 \qquad (A.31)$$

where following identities have been used

$$d_1' = c_4 b_2 - c_2 b_4 \qquad (A.32)$$

$$d_2' = c_1 b_4 - c_4 b_1 \qquad (A.33)$$

$$d_3' = c_2 b_1 - c_1 b_2 \qquad (A.34)$$

$$d_4' = (d_1' + d_2' + d_3') \qquad (A.35)$$

p', q', r', s' are similar to p, q, r, s as in (A.26) - (A.29), where d_i is replaced by d_i'.

Recalling that $e^{jx} = \text{Cos } x + i \text{ Sin } x$, we obtain real parts of equation (A.30) and (A.31), respectively as follows,

$$p_x \text{ Cos } (t_2-t_1) - p_y \text{ Sin } (t_2-t_1) + q_x \text{ Cos } t_1 - q_y \text{ Sin } t_1 + s_x \text{ Cos } t_2 - s_y \text{ Sin } t_2 + r_x = 0 \qquad (A.36)$$

$$p_x' \text{ Cos } (t_2-t_1) - p_y' \text{ Sin } (t_2-t_1) + q_x' \text{ Cos } t - q_y' \text{Sin } t_1 + s_x' \text{ Cos } t_2 - s_y' \text{ Sin } t_2 + r_x' = 0 \qquad (A.37)$$

These two equations are real and independent, transcendental in the two unknowns, t_1 and t_2. To transform these into polynomial equations, we utilize the following trigonometric identities,

$$\text{Cos } \theta = \frac{1-\tan^2 \frac{\theta}{2}}{1+\tan^2 \frac{\theta}{2}} \tag{A.38}$$

and

$$\text{Sin } \theta = \frac{2\tan \frac{\theta}{2}}{1+\tan^2 \frac{\theta}{2}} \tag{A.39}$$

To substitute these identities in equation (A.36), we first simplify the notation by letting

$$T_j = \tan \frac{t_j}{2} \quad (j = 3,4) \tag{A.40}$$

Thus equation (A.36) becomes

$$\begin{aligned}
p_x &\left[\frac{1-T_1^2}{1+T_1^2} \cdot \frac{1-T_2^2}{1+T_2^2} + \frac{4T_1T_2}{(1+T_1^2)(1+T_2^2)} \right] \\
&-p_y \left[\frac{2T_2(1-T_1^2)}{(1+T_1^2)(1+T_2^2)} - \frac{2T_1(1-T_2^2)}{(1+T_1^2)(1+T_2^2)} \right] \\
&+q_x \left[\frac{1-T_1^2}{1-T_1^2} \right] - q_y \left[\frac{2T_1}{1+T_1^2} \right] + s_x \left[\frac{1-T_2^2}{1+T_2^2} \right] - s_y \frac{2T_2}{1+T_2^2} \\
&+ r_x = 0
\end{aligned} \tag{A.41}$$

Equation (A.37) will give similar result; the difference, however will be the presence of primes (').

Equation (A.41) gives

$$T_2^{\,2}(A_1T_1^{\,2} + B_1T_1 + C_1) + T_2(A_5T_1^{\,2} + B_5T_1 + C_5)$$
$$+ (A_3T_1^{\,2} + B_3T_1 + C_3) = 0 \qquad (A.42)$$

Similarly equation (A.37) will yield

$$T_2^{\,2}\ (A_2T_1^{\,2} + B_2T_1 + C_2) + T_2(A_6T_1^{\,2} + B_6T_1 + C_6)$$
$$+(A_4\ T_1^{\,2} + B_4T_1 + C_4) = 0 \qquad (A.43)$$

where

$$A_1 = p_x - q_x + r_x - s_x \qquad (A.44)$$
$$B_1 = -2(p_y + q_y) \qquad (A.45)$$
$$C_1 = q_x - p_x + r_x - s_x \qquad (A.46)$$
$$A_3 = r_x + s_x - p_x - q_x \qquad (A.47)$$
$$B_3 = 2(p_y - q_y) \qquad (A.48)$$
$$C_3 = p_x + q_x + r_x + s_x \qquad (A.49)$$
$$A_5 = 2(p_y - s_y) \qquad (A.50)$$
$$B_5 = 4p_x \qquad (A.51)$$
$$C_5 = -2(p_y+s_y) \qquad (A.52)$$

Expressions for A, B and C with the subscripts 2, 4, 6 are in terms of p_x', q_x', s_y' etc., and are similar to expressions shown previously.

Applying Sylvester's dyalitic method of elimination, we multiply both equations (A.42) and (A.43) by T_2 and consider T_2, $T_2^{\,2}$ and $T_2^{\,3}$ as different variables. This yields a system of four linear equations with unknowns T_2, $T_2^{\,2}$ and $T_2^{\,3}$ (e.g. X, Y, Z) with coefficients in terms of unknown T_1.

In order to get a nontrivial solution, we must have

$$\begin{vmatrix} 0 & A_1T_1^2+B_1T_1+C_1 & A_5T_1^2+B_5T_1+C_5 & A_3T_1^2+B_3T_1+C_3 \\ 0 & A_2T_1^2+B_2T_1+C_2 & A_6T_1^2+B_6T_1+C_6 & A_4T_1^2+B_4T_1+C_4 \\ A_1T_1^2+B_1T_1+C_1 & A_5T_1^2+B_5T_1+C_5 & A_3T_1^2+B_3T_1+C_3 & 0 \\ A_2T_1^2+B_2T_1+C_2 & A_6T_1^2+B_6T_1+C_6 & A_4T_1^2+B_4T_1+C_4 & 0 \end{vmatrix} = 0 \tag{A.53}$$

This eliminant is the compatibility equation, which must be satisfied by T_1.

This determinant set equal to zero and expanded will yield an eighth degree polynomial equation in T_1.

$$R_8T_1^8+ R_7T_1^7+ R_6T_1^6+ R_5T_1^5+ R_4T_1^4+ R_3T_1^3+ R_2T_1^2+ R_1T_1+ R_0=0 \tag{A.54}$$

Where R_j's are given as follows :

$$R_8 = D_1E_1-F_1^2 \tag{A.55}$$

$$R_7 = D_2E_1+D_1E_2-2F_1F_2 \tag{A.56}$$

$$R_6 = D_1E_3+D_3E_1+D_2E_2-2F_1F_3-F_2^2 \tag{A.57}$$

$$R_5 = D_1E_4+D_4E_1+D_3E_2+D_2E_3-2F_1F_4-2F_2F_3 \tag{A.58}$$

$$R_4 = D_1E_5+D_5E_1+D_2E_4+ D_4E_2+D_3E_3-2F_1F_5-2F_2F_4-F_3^2 \tag{A.59}$$

$$R_3 = D_2E_5+D_5E_2+D_3E_4+D_4E_3-2F_2F_5-2F_3F_4 \tag{A.60}$$

$$R_2 = D_3E_5+D_4E_4+D_5E_3-2F_3F_5-F_4^{\,2} \tag{A.61}$$

$$R_1 = D_4E_5+D_5E_4-2F_4F_5 \tag{A.62}$$

$$R_0 = D_5E_5-F_5^{\,2} \tag{A.63}$$

Where D's, E's and F's are given as follows :

$$D_1 = A_4A_5-A_6A_3 \tag{A.64}$$

$$D_2 = A_4B_5+A_5B_4-A_3B_6-A_6B_3 \tag{A.65}$$

$$D_3 = A_4C_5+A_5C_4+B_4B_5-A_3C_6-A_6C_3-B_3B_6 \tag{A.66}$$

$$D_4 = B_4C_5+B_5C_4-B_3C_6-B_6C_3 \tag{A.67}$$

$$D_5 = C_4C_5-C_3C_6 \tag{A.68}$$

$$E_1 = A_1A_6-A_2A_5 \tag{A.69}$$

$$E_2 = A_1B_6+A_6B_1-A_2B_5-A_5B_2 \tag{A.70}$$

$$E_3 = A_1C_6+A_6C_1+B_1B_6-A_2C_5-A_5C_2-B_2B_5 \tag{A.71}$$

$$E_4 = B_1C_6+B_6C_1-B_2C_5-B_5C_2 \tag{A.72}$$

$$E_5 = C_1C_6-C_2C_5 \tag{A.73}$$

$$F_1 = A_1A_4-A_2A_3 \tag{A.74}$$

$$F_2 = A_1B_4+A_4B_1-A_3B_2-A_2B_3 \tag{A.75}$$

$$F_3 = A_1C_4+A_4C_1+ B_1B_4-A_3C_2-A_2C_3-B_3B_2 \qquad (A.76)$$

$$F_4 = B_4C_1+B_1C_4-B_3C_2 -B_2C_3 \qquad (A.77)$$

$$F_5 = C_1C_4-C_2C_3 \qquad (A.78)$$

This equation yields 8 roots which are complex pairs and/ or real roots. For each real root of T_1 we can easily find t_1,

$$t_3 = 2 \tan^{-1}(T_3) \qquad (A.79)$$

To obtain t_2, we return to equations (A.42) and (A.43). If these have common real root(s) for T_2, then

$$t_2 = 2 \tan^{-1} (T_2) \qquad (A.80)$$

To find t_3 and t_4 we use

$$t_3 = \frac{1}{j} \operatorname{Log}\left[\frac{d_1 e^{jt_1} + d_2 e^{jt_2} + d_4}{-d_3}\right] \qquad (A.81)$$

$$t_4 = \frac{1}{j} \operatorname{Log}\left[\frac{d_1' e^{jt_1} + d_2' e^{jt_2} + d_4'}{-d_3'}\right] \qquad (A.82)$$

Z_5, Z_2 and Z_6 can be found from (A.15), (A.11) and (A.6).

Similarly Z_1, Z_4 and Z_3 can be found as Z_5, Z_2 and Z_6, only difference being

$$\left.\begin{aligned} a_i &= 1 - e^{ju_i} & (A.83)\\ b_i &= e^{jv_i}-1 & (A.84)\\ c_i &= Z_2 - Z_2\, e^{jt_i} & (A.85)\end{aligned}\right\} \quad (i = 1,4)$$

APPENDIX B

FORCE MOMENT CONSTRAINTS

Free body diagram of the coupler link is shown in Figure 6.

The sum of the moments about point A must be zero.

$$
\begin{aligned}
\overset{\curvearrowright}{+} \quad \sum M_A = {} & W \cos\theta \cdot X_5 \sin(X_9+\eta) \\
& - W \sin\theta \cdot X_5 \cos(X_9+\eta) \\
& - \left[K_4 \cos(\pi-\delta-X_8) \quad X_3 \cdot \sin\eta\right] \\
& - \left[K_4 \sin(\pi-\delta-X_8) \quad X_3 \cdot \cos\eta\right]
\end{aligned} \tag{B.1}
$$

$$0 = X_5 \cdot W \cdot \sin(X_9+\eta-\theta) + X_3 \cdot K_4 \cdot \sin(\eta-\delta-X_8) \tag{B.2}$$

$$K_{4i} = \frac{X_5 \cdot W_i \cdot \sin(\eta_i + X_{9i} - \theta_i)}{X_3 \cdot \sin(X_{8i} + \delta_i - \eta_i)} \tag{B.3}$$

$$K_4^{\,2} - \sum_{i=1}^{n} K_{4i}^{\,2} \geqslant 0 \tag{B.4}$$

The sum of forces in the direction of $\bar{K}_2$ must be equal to zero.

$$\sum F = K_2 + \frac{\bar{W} \cdot \bar{K}_2}{K_2} + \frac{\bar{K}_4 \cdot \bar{K}_2}{K_2} \tag{B.5}$$

$$0 = K_2 + W \cos(X_7 + X_8 - \theta) + K_4 \cos(X_7 - \delta) \tag{B.6}$$

$$K_{2i} = -K_{4i} \cdot \cos(\delta_i - X_{7i}) - W_i \cos(\theta_i - X_{7i} - X_{8i}) \tag{B.7}$$

$$K_2^{\,2} - \sum_{i=1}^{n} K_{2i}^{2} \geqslant 0 \tag{B.8}$$

The sum of forces in the direction of vector '$\bar{t}$' must be equal to zero.

$$\sum F = \frac{T_2}{X_2} + \frac{\bar{W}.\bar{t}}{\left(\frac{T_2}{X_2}\right)} + \frac{\bar{K}_4.\,\bar{t}}{\left(\frac{T_2}{X_2}\right)} \tag{B.9}$$

$$0 = \frac{T_2}{X_2} + W.\ \mathrm{Cos}\left(\frac{3\pi}{2} - X_7 - X_8 + \theta\right) + K_4.\mathrm{Cos}\left(\frac{\pi}{2} + X_7 - \delta\right) \tag{B.10}$$

The vector $\bar{t}$ is defined such that $\bar{X}_2 \times \bar{t}$ gives the moment $\bar{T}_2$.

$$T_{2i} = -X_2 \cdot K_{4i} \cdot \mathrm{Sin}\,(\delta_i - X_{7i}) - X_2 \cdot W_i \cdot \mathrm{Sin}\,(\theta_i - X_{7i} - X_{8i}) \tag{B.11}$$

$$T^2 - \sum_{i=1}^{n} T_{2i}^2 \geq 0 \tag{B.12}$$

APPENDIX C

CHOICE OF INITIAL VALUE OF 'r'

Two methods to choose the initial value of 'r' have been suggested in the literature.

(1) $$r_1 = \frac{\nabla f(\bar{X}_0)^T \cdot \nabla p(\bar{X}_0)}{|\nabla p(\bar{X}_0)|^2} \qquad (C.1)$$

where $$p(\bar{X}_0) = \sum_{i=1}^{m} \frac{1.0}{g_i(\bar{X}_0)} \qquad (C.2)$$

and represents the gradient of the function.

CASE I : $r_1 < 0$

If $r_1 < 0$, minimization of $f(\bar{X}_0)$ alone, without considering the penalty term, is carried out. At every new point the r-value is checked by equation (C.1). If the value is positive, unconstrained minimization with obtained value of r is carried out.

CASE II : $r_1 = 0$

This means that the unconstrained minimum has been reached $\bar{X}_0 = \bar{X}$.

CASE III : $r_1 > 0$

If $r_1 > 0$, this is taken as the starting value for usual minimization. That value of r_1 is selected

which makes the objective function and Penalty term equal

$$r_1 = \frac{f(\bar{X}_0)}{\sum_{i=1}^{m} \frac{1.0}{g_i(\bar{X})}} \tag{C.3}$$

This has two weaknesses. Firstly, the starting point may be too close to some of the boundaries thereby making such selection of r useless. Secondly, there may not be a point in the feasible domain which is not near a constraint boundary. If some of the constraints are near the boundary, i.e. penalty is large, the value of 'r' obtained from (C.3) is very small. This difficulty is eliminated if in the summation for penalty, these constraints which are quite close to the boundary are substituted by some minimum value of constraint modified expression for r_1 is

$$r_1 = \frac{f(\bar{X}_0)}{\sum_{i=1}^{m} \frac{1}{\max\left[g_i(\bar{X}_0),\ g_{min}\right]}} \tag{C.4}$$

Since constraints are normalized here, the value of g_{min} can be arbitrarily selected as 0.2.

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```
$JOB  MEG134,TIME008,PAGES01/,NAME S.K.CHATURVEDI
$EXECUTE        WATFOR,REWIND
C      ** OPTIMIZATION PROGRAMME FOR FOUR-BAR LINK GUIDING PROBLEM**
C      XP(I) CORRESPONDS TO DESIRED X-COORDINATE OF COUPLER POINT
C      YP(I) CORRESPONDS TO DESIRED Y-COORDINATE OF COUPLER POINT
C      W(I) CORRESPONDS TO FORCE APPLIED AT COUPLER POINT AT AN ANGLE THETA(I)
C      WITH THE HORIZONTAL
C      DETA(I) GIVES SUCCESSIVE ANGULAR INCREMENTS IN COUPLER POSITIONS
C      DGAMA(I) GIVES SUCCESSIVE ANGULAR INCREMENTS IN CRANK
C      N GIVES NUMBER OF DESIGN  VARIABLES IN DESIGN VECTOR
C      NIN GIVES NUMBER OF CONSTRAINTS
C      NPOS GIVES NUMBER OF POSITION SYNTHESIS
C      FEASIBLE REGION FOR THE PIVOT A IS DEFINED WITH RADIUS RA AROUND THE
C      POINT FIXED BY RADIUS VECTOR XA WITH ANGULAR POSITION ALA
C      XB, RB, AND ALB ARE ANALOGOUS TO XA, RA, AND ALA FOR PIVOT A
C      **** X(I) IS THE STARTING SOLUTION OF THE PROBLEM ****
       COMMON/P1/PI
       COMMON/MANE/N
       COMMON/INEQLT/CON(30),SATSFY
       COMMON/SUB1/DETA(6),DGAMA(6),XP(7),YG(7)
       COMMON/SUB2/XLMIN,XLMAX
       COMMON/SUB3/NPOS,NIN
       COMMON/SUB4/SIGMA
       COMMON/SUB5/W(7),THETA(7),RA,XA,ALA,RB,XB,ALB,DESRT2,DESRK4,DESRK2
       SIGMA=1.
       DIMENSION X(10)
       PI=4.*ATAN(1.)
       N=10
       NIN=26
       NPOS = 7
       NPOS1=NPOS-1
       DATA DETA/-12.,-7.,-1.,1.,6.,5./,DGAMA/40.,40.,40.,20.,20.,20./
       DATA XP/10.9T9.6,7.3,5.6,5.3,5.3,5.6/,YG/11.7,12.8,11.9,9.7,8.3,
      17.1,6.0/
       DATA X/12.,4.,9.5,8.,4.,8.5,31.,11.,29.,36./
       DATA W/7*10./,THETA/7*10./
       DATA XA,RA,ALA,XB,RB,ALB,DESRT2,DESRK4,DESRK2/8.,4.,30.,20.,4.,
      125.,50.,20.,30./
       ALA=ALA*PI/180.
       ALB=ALB*PI/180.
       DO 28 I=1,NPOS
 28    THETA(I)=THETA(I)*PI/180.
       DO 26 I=1,NPOS1
       DETA(I)=DETA(I)*PI/180.
 26    DGAMA(I)=DGAMA(I)*PI/180.
       DO 27 I=7,10
 27    X(I)=X(I)*PI/180.
       PRINT 50
       PRINT 100,(XP(I),I=1,NPOS)
       PRINT 51
```

```
      PRINT 100,(YG(I),I=1,NPOS)
      PRINT 52
      PRINT 100,(DGAMA(I),I=1,NPOS1)
      PRINT 53
      PRINT 100,(DETA(I),I=1,NPOS1)
      XLMIN=1.
      XLMAX=20.
      CALL MINIMA(X)
   50 FORMAT(10X,*XP(I)*)
   52 FORMAT(10X,*DGAMA(I)*)
   51 FORMAT(10X,*YG(I)*)
   53 FORMAT(10X,*DETA(I)*)
  100 FORMAT(10X,8F15.4)
      STOP
      END
$IBFTC MINIMA
      SUBROUTINE MINIMA(X)
      COMMON/MANE/N
      COMMON/MINI/R
      COMMON/POVEL/Z
      COMMON/SERCH2/FUNVAL
      COMMON/INEQLT/CON(30),SATSFY
      DIMENSION X(10),X1(10)
      R=10.
  281 I=0
   52 CALL FUNT(X,Z)
      IF(I.NE.0)GO TO 53
      PRINT 100,FUNVAL
      CALL OUTPUT(X)
      PRINT 70,(CON(J),J=1,23)
 70   FORMAT(21X,10F11.4'
      CALL CHECK(X)
      IF(SATSFY.EQ.0.) GO TO 53
      PRINT 59
 59   FORMAT(//20X,*INITIAL SOLUTION DOES NOT LIE ON CONSTRAINT SET*)
      STOP
   53 PRINT 600
      PRINT 500
      PRINT 650 ,R
      PRINT 600
      CALL POWELL(X)
      I=I+1
      IF(I.EQ.1)GO TO 80
      DIFF=ABS(Z-Z1)
      IF(DIFF.LE.ABS(Z*1.E-06))GO TO 90
   80 Z1=Z
      R=R/4.
      GO TO 52
  100 FORMAT(//10X,*INITIAL FUNCTIONVALUE*,E15.6,10X,*SOLUTION VECTOR*)
  500 FORMAT(6X,*R*,9X,*ITRN.*, 5X,*SECONDARY*,6X,*OBJECTIVE *,20X,*SOLU
     1TION VECTOR*/26X,*FUNCTION*,7X,*FUNCTION*)
```

```
C    VALUE OF  ALPHA =0 IN LINEAR MIN. MEANS NO PROGRESS IN LINEAR MIN.
      IF(ITN.EQ.0)GO TO 920
      IF(Z.LT.Z1) GO TO 920
C  THIS FOLLOWS F(X0+ALPHA*S) = F0
      Z=Z1
      RETURN
  920 DO 92 I=1,N
      SIGMA(I)=ALPI*S(I)
      ET=AMAX1(1.,ABS(ALPI))
      DO 76 L=1,N
      IF(ET*S(L).LE.EPSLON(L)) GO TO 76
      GO TO 89
 76   CONTINUE
      RETURN
 89   DOTSIG=DOTSIG+SIGMA(I)**2
   92 X(I)=X(I)+SIGMA(I)
      CALL GRAD(X,G1)
      AR=0
      BR=0
      CR=0
      DR=0
      DO 93 I=1,N
      AR=AR+G1(I)*G1(I)
      BR=BR+S(I)*S(I)
      DR=DR+G1(I)*S(I)
      Y(I)=G1(I)-G(I)
      CR=CR+Y(I)*S(I)
 93   G(I)=G1(I)
      DO 86 J=1,N
      RO(J)=RO(J)+(1./(ALPI*CR)-DR/(ALPI*CR)**2 )*Y(J)**2+2./CR*(Y(J)*G
     1(J))
      IF(RO(J).LE.0.) GO TO 71
 86   CONTINUE
      A1=0.
      A2=0.
      DO 95 I=1,N
      A1=A1+SIGMA(I)*Y(I)
      DO 95 J=1,N
   95 A2=A2+Y(I)*H(I,J)*Y(J)
      A1=1./A1
      A2=1./A2
      DO 98 I=1,N
      DO 98 J=1,N
      H1(I,J)=H(I,J)+SIGMA(I)*SIGMA(J)*A1
      DO 98 K=1,N
      DO 98 M=1,N
   98 H1(I,J)=H1(I,J)+A2*H(I,K)*Y(K)*Y(M)*H(M,J)
      DO 99 I=1,N
      DO 99 J=1,N
   99 H(I,J)=H1(I,J)
      ITN=ITN+1
```

```
      PRINT 5000,ITN,Z,F
      CALL OUTPUT(X)
      IF(ITN.EQ.10*N) GO TO 106
  100 Z1=Z
      GO TO 90
 106  PRINT 107
 107  FORMAT(///10X,* MAXIMUM NO OF ITERATIONS EXCEEDED( HENCE QUIT*)
 5000 FORMAT(16X I3,2X,2F15.6)
  105 RETURN
      END
$IBFTC GRAD
      SUBROUTINE GRAD(X,G)
      COMMON/MANE/N
      COMMON/SUB3/NPOS,NIM
      COMMON/INECLT/CON(30),SATSFY
      COMMON/POVEL/Z
      DIMENSION X(10),X1(10),G(10)
      FX=Z
      DO 100 I=1,N
      DO 10 J=1,N
   10 X1(J)=X(J)
      DX=X(I)*.1
      IF(DX.EQ.0.) DX=.01
    5 X1(I)=X(I)+DX
      CALL CHECK(X1)
      IF(SATSFY.EQ.0.)GO TO 20
      IF(DX.LT.0.)GO TO 11
      DX=-DX
      GO TO 5
   11 DX=-DX*.1
      GO TO 5
   20 CALL FUNT(X1,FX1)
  100 G(I)=(FX1-FX)/DX
      RETURN
      END
$IBFTC FUNXON
      FUNCTION FUNXON(X)
      COMMON/SUB4/SIGMA
      COMMON/PI/PI
      COMMON/MANE/N
      COMMON/INECLT/CON(30),SATSFY
      COMMON/SUB1/DETA(6),DGAMA(6),XP(7),YG(7)
      COMMON/SUB3/NPOS,NIM
      COMMON/ANG/ETA,GAMA
      COMMON/SUBS/I
      DIMENSION X(10)
      FUNXON=0.
      GAMA=X(7)
      DO 100 I=1,NPOS
      CALL ANGLE(X)
      GAMA=GAMA+X(8)
      ETA=ETA+X(9)
```

```
      XA=X(6)*COS(X(10))+X(2)*COS(GAMA)+X(5)*COS(ETA)
      YA=X(6)*SIN(X(10))+X(2)*SIN(GAMA)+X(5)*SIN(ETA)
      FUNXON=FUNXON+(XA-XP(I))**2+(YA-YG(I))**2
      GAMA=GAMA-X( )
      ETA=ETA-X(9)
      IF(I.EQ.1)GO TO 90
      DETA1=ETA-ETA1
      IF(DETA1.GE.2.*PI)DETA1=DETA1-2.*PI
      IF(DETA1.LE.(-2.)*PI) DETA1=DETA1+2.*PI
      FUNXON=FUNXON+SIGMA*(DETA1-DETA(I-1))**2
   90 ETA1=ETA
      IF(I.EQ.NPOS)GO TO 100
      GAMA=GAMA+DGAMA(I)
  100 CONTINUE
      RETURN
      END
$IBFTC FUNT
      SUBROUTINE FUNT(X,FX)
      COMMON/MINI/R
      COMMON/SUB3/NPOS,NIN
      COMMON/INEQLT/CON(30),SATSFY
      COMMON/MANE/N
      COMMON/SERCH2/FUNVAL
      DIMENSION X(10)
      FX=FUNXON(X)
      FUNVAL=FX
      CALL CONSTR(X)
      DO 10 I=1,NIN
   10 FX=FX+R/CON(I)
      RETURN
      END
$IBFTC CHECK
      SUBROUTINE CHECK(X)
      COMMON/MANE/N
      COMMON/SUB3/NPOS,NIN
      COMMON/INEQLT/CON(30),SATSFY
      COMMON/SUB2/XLMIN,XLMAX
      COMMON/SUB1/DETA(6),DGAMA(6),XP(7),YG(7)
      DIMENSION X(10)
      SATSFY=0
      CALL CONSTR(X)
      DO 10 I=1,NIN
      IF(CON(I).LE.0.)GO TO 15
   10 CONTINUE
      RETURN
   15 SATSFY=1
      RETURN
      END
$IBFTC ANGLE
      SUBROUTINE ANGLE(X)
```

```
      COMMON/ANG/ETA,GAMA
      COMMON/P1/PI
      COMMON/SUB4/I
      DIMENSION X(10)
      COMMON/SUB5/W(7),THETA(7),RA,XA,ALA,RB,XB,ALB,DESRT2,DESRK4,DESRK2
      COMMON/CONST/ T2,K4,K2
      REAL K2,K4
      K4=0
      K2=0
      T2=0
      D=X(1)**2+X(2)**2-2.*X(1)*X(2)*COS(GAMA)
      D=SQRT(D)
      TEMP=(X(2)*SIN(GAMA)/D)
      XLMD=ARSIN(TEMP)
      COSBET=(D*D+X(4)**2-X(3)**2)/(2.*D*X(4))
      BETA=ARCOS(COSBET)
      DELTA=PI-BETA-XLMD
      X3X=X(1)*COS(X(8))+X(4)*COS(DELTA+X(8))-X(2)*COS(GAMA+X(8))
      X3Y=X(1)*SIN(X(8))+X(4)*SIN(DELTA+X(8))-X(2)*SIN(GAMA+X(8))
      IF(ABS(X3X).LE.1.E-38)GO TO 100
      TEMP=ABS(X3Y/X3X)
      TEMP=ATAN(TEMP)
      IF((X3X.GT.0.).AND.(X3Y.LT.0.))TEMP=-TEMP+2.*PI
      IF((X3X.LT.0.).AND.(X3Y.GT.0.))TEMP=PI-TEMP
      IF((X3X.LT.0.).AND.(X3Y.LT.0.))TEMP=PI+TEMP
      ETA=TEMP
      K4=K4+(X(5)*W(I)*SIN(ETA+X(9)-THETA(I)))/(X(3)*SIN(X(8)+DELTA-ETA)
     1)
      T2=T2-X(2)*K4*SIN(DELTA-GAMA)-X(2)*W(I)*SIN(THETA(I)-GAMA-X(8))
      K2=K2-K4*COS(DELTA-GAMA)-W(I)*COS(THETA(I)-GAMA-X(8))
      RETURN
  100 IF(X3Y.LT.0.)ETA=-PI/2.
      IF(X3Y.GE.0.)ETA=+PI/2.
      K4=K4+(X(5)*W(I)*SIN(ETA+X(9)-THETA(I)))/(X(3)*SIN(X(8)+DELTA-ETA)
     1)
      T2=T2-X(2)*K4*SIN(DELTA-GAMA)-X(2)*W(I)*SIN(THETA(I)-GAMA-X(8))
      K2=K2-K4*COS(DELTA-GAMA)-W(I)*COS(THETA(I)-GAMA-X(8))
      RETURN
      END
$IBFTC CONSTR
      SUBROUTINE CONSTR(X)
      COMMON/INEQLT/CON(30),SATSFY
      COMMON/P1/PI
      COMMON/SUB2/YLMIN,YLMAX
      COMMON/SUB3/NPOS,NIN
      COMMON/SUB5/W(7),THETA(7),RA,XA,ALA,RB,XB,ALB,DESRT2,DESRK4,DESRK2
      REAL K2,K4
      DIMENSION X(10)
      COMMON/CONST/ T2,K4,K2
      PRINT,(X(I),I=1,10)
      DO 10 I=1,4
      CON(I)=X(I)-XLMIN
   10 CON(I+4)=XLMAX-X(I)
      CON(9)=X(3)+X(4)-X(1)-X(2)
      CON(10)=(X(1)-X(2))**2-(X(3)-X(4))**2
      DO 15 I=7,10
      CON(I+4)=X(I)
   15 CON(I+8)=2.*PI-X(I)
      CON(19)=X(1)-X(2)
      CON(20)=X(3)-X(2)
      CON(21)=X(4)-X(2)
```

```
      CON(22)=RB**2-X(6)**2-X(1)**2-XB**2-2.*X(1)*X(6)*COS(X(10)-X(8))
     1+2.*XB*X(6)*COS(ALB-X(10))+2.*XB*X(1)*COS(ALB-X(8 ))
      CON(23)=RA**2-XA**2-X(6)**2+2.*XA*X(6)*COS(ALA-X(10))
      RETURN
      END
FTC OUTPUT
      SUBROUTINE OUTPUT(X)
      COMMON/NAME/N
      DIMENSION X(10)
      PRINT 100,(X(I),I=1,N)
0     FORMAT(56X,5E15.6///////)
      RETURN
      END
FTC BETA
      SUBROUTINE BETA(X,S,ALPHA)
      COMMON/CODE/ICD
      COMMON/SERCH1/EMIN,EMAX,EMID
      COMMON/NAME/N
      COMMON/SUB/NIN
      COMMON/INECLT/CON(30),SATSFY
      DIMENSION X(10),S(10),Y(10)
      ETA=2.
      J=0
00    DO 110 I=1,N
10    Y(I)=X(I)+ETA*S(I)
      CALL CHECK(Y)
      IF(SATSFY.EQ.0.)GO TO 93
      J=1
      ETA=ETA/2.
      GO TO 100
93    IF(J.NE.0)GO TO 94
      ETA=2.*ETA
      GO TO 100
94    EMIN=ETA
      EMAX=2.*ETA
      ICD=1
      CALL SEARCH(X,S)
      EMIN=0
      EMAX=EMID
      ICD=2
      CALL SEARCH(X,S)
      ALPHA=EMID
      RETURN
      END
FTC SEARCH
      SUBROUTINE SEARCH(X,S)
      COMMON/SERCH1/EMIN,EMAX,EMID
      COMMON/CODE/ICD
      COMMON/SERCH4/EMIN1,EMAX1
      COMMON/POVEL/Z
      COMMON/SERCH2/FUNVAL
      COMMON/SERCH3/F
      EMIN1=EMIN
      EMAX1=EMAX
      CONST1=0.381966
```

```
      CONST2=0.618034
      DIMENSION X(10),S(10)
   10 DIST=EMAX-EMIN
      DE=DIST
   11 E1=EMIN+CONST1*DE
      E2=EMIN+CONST2*DE
      CALL FUN1(X,S,E1,F1)
      FU1=FUNVAL
      CALL FUN1(X,S,E2,F2)
      FU2=FUNVAL
   12 IF(ABS(EMAX-EMIN).LE..001*ABS(DIST)) GO TO 30
      DE=CONST2*DE
      IF(F1-F2)13,25,14
   14 EMIN=E1
      E1=E2
      F1=F2
      FU1=FU2
      E2=EMIN+CONST2*DE
      CALL FUN1(X,S,E2,F2)
      FU2=FUNVAL
      GO TO 12
   13 EMAX=E2
      F2=F1
      E2=E1
      FU2=FU1
      E1=EMIN+CONST1*DE
      CALL FUN1(X,S,E1,F1)
      FU1=FUNVAL
      GO TO 12
   25 IF(ICD.EQ.1)GO TO 13
      EMIN=E1
      EMAX=E2
      DE=EMAX-EMIN
      GO TO 11
   30 IF(F1.GT.F2)GO TO 32
      EMID=E1
      Z=F1
      F=FU1
      RETURN
   32 EMID=E2
      Z=F2
      F=FU2
      RETURN
      END
SIBFTC FUN1
      SUBROUTINE FUN1(X,S,E,F)
      COMMON/NAME/N
      COMMON/SERCH4/EMIN1,EMAX1
      COMMON/CODE/ICD
      COMMON/INEQLT/CON(30),SATSFY
      DIMENSION X(10),S(10),Y(10)
      DO 10 I=1,N
   10 Y(I)=X(I)+E*S(I)
```

```
      IF(ICD.FQ.1)GO TO 50
      CALL FUNT(Y,F)
      RETURN
   50 CALL CHECK(Y)
      IF(SATSFY.FQ.1.) GO TO 60
      F=100.*(1.-((E-EMIN1)/(EMAX1-EMIN1))**2)
      RETURN
11 60 F=1.F+.5
      RETURN
      END
 $ENTRY
```

```
*JOB  MEG-34,TIME008,PAGES010,NAME S.K.CHATURVEDI
C***
C***
C***KINEMATIC SYNTHESIS OF 4-BAR LINKAGE FOR 5-POSITION LINK GUIDI
C***
$IBJOB
$IBFTC MAIN
      COMPLEX ONE,F,B(4),C(4),C1(3),ET3(2),ET4(2),EV3(2),EV4(2),Z:
     1Z3,Z4,Z5,Z6
      DIMENSION T1(8),RR(8),RI(8),T2(2),V1(8),V2(2),G(9),U(4)
      DIMENSION X(5),Y(5)
      DOUBLE PRECISION ZR(100),ZI(100),S1(9)
      COMMON/MANE/C,B,T1,T2,V1,V2,X,Y,ONE
      COMMON/MAN/G1,G2,G3,G4,H1,H2,H3,H4
      DATA X/10.9,9.6,7.3,5.6,5.3/,Y/11.7,12.8,11.9,9.7,8.3/,U(1),
     1U(3),U(4)/-12.,-19.,-20.,-19./
      PI=4.*ATAN(1.)
      ONE=(1.,0.)
      DO 9 I=1,4
      U(I)=U(I)*PI/180.
      B(I)=F(1.,U(I))-ONE
      C(I)=CMPLX(XUI+1),Y(I+1))-CMPLX(X(1),Y(1))
      9F(I.EQ.1) GO TO 9
      31(I-1)=C(1)-C(I)/B(I)*B(1)
 9    CONTINUE
      3ALL SYNTH1(G,C1)
      PRINT 40,(G(I),I=1,9)
 40   FORMAT(1X,E15.8)
      CALL NORML(G)
      DO 21 I=1,9
 21   S1(I)=G(I)
      CALL POLY(8,S1,ZR,ZI)
      DO 12 I=1,8
      RR(I)=ZR(I)
 12   RI(I)=ZI(I)
      PRINT 10,(RR(I),I=1,8),(RI(I),I=1,8)
 10   FORMAT(///* REAL ROOTS*,5X/8F15.8//* IMAGINARY ROOTS*/8F15.(
      N=0
      DO 11 I=1,8
      IF((RI(I).LT.0.).OR.(RI(I).GT.0.)) GO TO 11
      N=N+1
      T1(I)=RR(I)
 11   CONTINUE
      DO 13 I=1,N
      CALL SYNTH2(I,M,T2,T1)
      T1(I)=2.*ATAN(T1(I))
      IF(M.EQ.0) GO TO 13
      DO 13 J=1,M
      ET3(J)=(G1*F(1.,T1(I))+G2*F(1.,T2(J))+G4)/G3
      ET4(J)=(H1*F(1.,T1(I))+H2*F(*.,T2(J))+H4)/H3
      CALL SYNTH4(I,J,Z2,Z5,Z6)
      C(1)=Z 2 *(F(1.,T1(I))-ONE)
      C(2)=Z 2 *(F(1.,T2(J))-ONE)
```

```
      C(3)=Z 2 *(ET3(J)-ONE)
      C(4)=Z 2 *(ET4(J)-ONE)
      DO 14 K=1,4
      B(K)=,B(K)
      IF(K.EQ.1) GO TO 14
      C1(K-1)=C(1)-C(K)/B(K)*B(1)
 14   CONTINUE
      3ALL SYNTH1(G,C1)
      PRINT 40,(G(IM),IM=1,9)
      CALL NORML(G)
      DO 22 IM=1,9
 22   S1(IM)=G(IM)
      CALL POLY(8,S1,ZR,ZI)
      DO 16 L=1,8
      RR(L)=ZR(L)
 16   RI(L)=ZI(L)
      PRINT 17,(RR(L),L=1,8),(RI(L),L=1,8)
 17   FORMAT(///* REAL ROOTS*,5X/8F15.8//* IMAGINARY ROOTS*/8F15.8)
      N1=0
      DO 18 L=1,8
      IF((RI(L'.LT.0.).OR.(RI(L).GT.0.)) GO TO 18
      N1=N1+1
      V1(L)=RR(L)
 18   CONTINUE
      DO 13 L=1,N1
      CALL SYNTH2(L,M1,V2,V1)
      V1(L)=2.*ATAN(V1(L))
      IF(M1.EQ.0) GO TO 13
      DO 13 K=1,M1
      EV3(K)=(G1*F(1.,V1(L))+G2*F(1.,V2(K))+G4)/G3
      EV4(K)=(H1*F(1.,V1(L))+H2*F(1.,V2(K))+H4)/H3
      CALL SYNTH5(L,K, Z4,Z3,Z1,Z2)
      PRINT 30,Z1,Z2,Z3,Z4,Z5,Z6
 30   FORMAT(//11X,6(F15.8,5X))
 13   CONTINUE
      STOP
      END
$IBFTC SYNTH1
      SUBROUTINE SYNTH1(G,Y1)
      DIMENSION G(9),T1(8),T2(2),V*(8),V2(2),X(3),Y(3)
      COMMON/MAN/G1,G2,G3,G4,H1,H2,H3,H4
      COMMON/MANE/C,B,T1,T2,V1,V2,X,Y,ONE
      COMMON/SOB2/A1,A2,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6
      COMPLEX P,Q,R,S,P1,Q1,R1,S1,ONE,Z2
      COMPLEX B(4),C(4),Y1(3),G1,G2,G3,G4,G5,H1,H2,H3,H4,H5
      G1=B(3)*B(2)*(ONE-Y1(1)/Y1(2))
      G2=-B(3)*B(1)
      G3=B(1)*B(2)*Y1(1)/Y1(2)
      G4=-(G1+G2+G3)
      G5=G1*CONJG(G1)+G2*CONJG(G2)+G4*CONJG(G4)-G3*CONJG(G3)
      H1=B(2)*B(4)*(ONE-Y1(1)/Y1(3))
      H2=-B(1)*B(4)
      H3=B(1)*B(2)*Y1(1)/Y1(3)
```

```
H4=-(H1+H2+H3)
H5=H1*CONJG(H1)+H2*CONJG(H2)+H4*CONJG(H4)-H3*CONJG(H3)
P=G1*CONJG(G2)
Q=G1*CONJG(G4)
S=G2*CONJG(G4)
R=G5
P1=H1*CONJG(H2)
Q1=H1*CONJG(H4)
S1=H2*CONJG(H4)
R1=H5
A1=REAL(P-Q+R-S)
B1=-2.*AIMAG(P+Q)
C1=REAL(Q-P+R-S)
A3=REAL(R+S-P-Q)
B3=2.*AIMAG(P-Q)
C3=REAL(P+Q+R+S)
A5=2.*AIMAG(P-S)
B5=4.*REAL(P)
C5=-2.*AIMAG(P+S)
A2=REAL(P1-Q1+R1-S1)
B2=-2.*AIMAG(P1+Q1)
C2=REAL(Q1-P1+R1-S1)
A4=REAL(R1+S1-P1-Q1)
B4=2.*AIMAG(P1-Q1)
C4=REAL(P1+Q1+R1+S1)
A6=2.*AIMAG(P1-S1)
B6=4.*REAL(P1)
C6=-2.*AIMAG(P1+S1)
D1=A4*A5-A6*A3
D2=A4*B5+A5*B4-A3*B6-A6*B3
D3=A4*C5+A5*C4+B4*B5-A3*C6-A6*C3-B3*B6
D4=B4*C5+B5*C4-B3*C6-B6*C3
D5=C4*C5-C3*C6
E1=A1*A6-A2*A5
E2=A1*B6+A6*B1-A2*B5-A5*B2
E3=A1*C6+A6*C1+B1*B6-A2*C5-A5*C2-B2*B5
E4=B1*C6+B6*C1-B2*C5-B5*C2
E5=C1*C6-C2*C5
F1=A1*A4-A2*A3
F2=A1*B4+A4*B1-A3*B2-A2*B3
F3=A1*C4+A4*C1+B1*B4-A3*C2-A2*C3-B3*B2
F4=B4*C1+B1*C4-B3*C2-B2*C3
F5=C1*C4-C2*C3
G(1)=D5*E5-F5**2
G(2)=D4*E5+D5*E4-2.*F4*F5
G(3)=D3*E5+D4*E4+D5*E3-2.*F3*F5-F4**2
G(4)=D2*E5+D3*E4+D4*E3+D5*E2-2.*F2*F5-2.*F3*F4
G(5)=D1*E5+D2*E4+D3*E3+D4*E2+D5*E1-2.*F1*F5-2.*F2*F4-F3**2
G(6)=D1*E4+D2*E3+D3*E2+D4*E1-2.*F1*F4-2.*F2*F3
G(7)=D1*E3+D3*E1+D2*E2-2.*F1*F3-F2**2
G(8)=D2*E1+D1*E2-2.*F1*F2
G(9)=D1*E1-F1**2
```

```
      RETURN
      END
$IBFTC SYNTH2
      SUBROUTINE SYNTH2(I,M,T2,T)
      COMMON/SUB2/A1,A2,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6
      DIMENSION T2(2),XR(2),XI(2),YR(2),YI(2),X(2),Y(2),RR(3),RI(3),T(8)
      RR(1)=FUNC(A3,B3,C3,I,T)
      RR(2)=FUNC(A5,B5,C5,I,T)
      RR(3)=FUNC(A1,B1,C1,I,T)
      RI(1)=FUNC(A4,B4,C4,I,T)
      RI(2)=FUNC(A6,B6,C6,T)
      RI(3)=FUNC(A2,B2,C2,T)
      CALL QUAD(RR,XR,XI)
      CALL QUAD(RI,YR,YI)
      M=0
      DO 14 J=1,2
      IF((XI(J).LT.0.).OR.(XI(J).GT.0.)) RETURN
      X(J)=XR(J)
 15   IF((YI(J).LT.0.).OR.(YI(J).GT.0.)) RETURN
      Y(J)=YR(J)
 14   CONTINUE
      DO 16 J=1,2
      IF((X(J).GT.Y(J)).OR.(X(J).LT.Y(J))) GO TO 16
      T2(J)=2.*ATAN( X(J))
      M=M+1
 16   CONTINUE
      IF(M.EQ.2) RETURN
      P=X(1)
      Q=X(2)
      DO 17 J=1,2
      IF((X(J).GT.Y(J)).OR.(X(J).LT.Y(J))) GO TO 17
      M=M+1
      T2(J)=2.*ATAN(X  (J))
 17   CONTINUE
      RETURN
      END
$IBFTC SYNTH4
      SUBROUTINE SYNTH4(I,J,Z2,Z5,Z6)
      COMMON/MANE/C,B,T1,T2,V1,V2,X,Y,ONE
      DIMENSION X(5),Y(5),T1(8),T2(2),V1(8),V2(2)
      COMPLEX Z6,Z2,Z5,C11,A(4),B(4),C(3),F,ONE,A11
      A(1)=F(1.,T1(I))-ONE
      A(2)=F(1.,T2(J))-ONE
      A11=A(1)-A(2)/B(2)*B(1)
      C11=C(1)-C(2)/B(2)
      Z5   =C11/A(1)
      Z2   =(C(1)-B(1)*Z 5 )/A(1)
      Z6   =F(X(1),Y(1))-Z2-Z5
      RETURN
      END
$IBFTC SYNTH5
      SUBROUTINE SYNTH5(L,K,Z4,Z3,Z1,Z2)
      COMMON/MANE/C,B,T1,T2,V1,V2,X,Y,ONE
```

```
      DIMENSION X(5),Y(5),T1(8),V1(8),T2(2),V2(2)
      COMPLEX B(4),C(3),Z1,Z2,Z3,Z4,C11,F,ONE,A(4),A11
      A(1)=F(1.,V1(L))-ONE
      A(2)=F(1.,V2(K))-ONE
      A11=A(1)-A(2)/B(2)*B(1)
      C11= C(1)-C(2)/B(2)
      Z3=C11/A(1)
      Z4=(C(1)-B(1)*Z5)/A(1)
      Z1=Z2+Z3-Z4
      RETURN
      END
$IBFTC FUNC
      FUNCTION FUNC(P,Q,S,T)
      COMMON/MANE/C,B,T1,T2,V1,V2,X,Y,ONE
      DIMENSION T1(8),T2(2),V1(8),V2(2),X(5),Y(5),T(9)
      COMPLEX C(3),B(4),ONE,Z2
      FUNC=P*T(I)**2+Q*T(I)+S
      RETURN
      END
$IBFTC QUAD
      SUBROUTINE QUAD(R,X,Y)
      DIMENSION R(3),X(2),Y(2)
      X(1)=-R(2)/(2.*R(1))
      X(2)=X(1)
      RADCL=R(2)**2-4.*R(1)*R(3)
      IF(RADCL) 20,21,22
 20   Y(1)=SQRT(-RADCL)/(2.*R(1))
      Y(2)=-Y(1)
      RETURN
 21   Y(1)=0
      Y(2)=0
 22   X(1)=X(1'+SQRT(RADCL)/(2.*R(1))
      X(2)=X(1)-SQRT(RADCL)/(2.*R(1))
      Y(1)=
      Y(2)=0
      RETURN
      END
$IBFTC F
      COMPLEX FUNCTION F(X,Y)
      F=CMPLX(X,0.)*CMPLX(COS(Y),SIN(Y))
      RETURN
      END
$IBFTC NORML
      SUBROUTINE NORML(G'
      DIMENSION G(9)
      X=ABS(G(1))
      DO 10  I=2,9
      IF(X.GT.ABS(G(I)))  X=G(I)
  100 CONTINUE
      DO 101 I=1,9
  101 G(I)=G(I)/X
      RETURN
```

```
      END
FETCH POLY     MEG134
   C  POLY IS LIBRARY SUBROUTINE AT I.I.T. KANPUR COMPUTER CENTRE
   C  AND HAS BEEN CALLED FROM THE DISK
   C  SUBROUTINE POLY(NN,CC,RR,RI)
   C  MAIN LINE FOR COMPUTING ROOTS,R, OF THE POLYNOMIAL,
   C  CC(1)+CC(2)*X+CC(3)*(X**2)+ ---------------- +CC(N+1)*(X**NN)
   C  ROOTS ARE COMPUTED FROM THE COFFICIENTS VIA THE MULLER METHOD REFINED BY
   C  NEWTONS METHOD.
   C  RR(NN) ARE THE NN REAL ROOTS OF THE POLYNOMIAL.
   C  RI(NN) ARE THE NN IMAGINARY ROOTS OF THE POLYNOMIAL.
      DOUBLE PRECISION TP1,RR(100),RI(100),CC(9),CCC(101),SR(6)          POL001T
DONE
ENTRY
```